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UNSTEADY HEAT TRANSFER ANALYSIS FOR CHOSEN AMMUNITION AND GUN

Rao V. S. Yalamanchili

Army Weapons Command
Rock Island, Illinois

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TECHNICAL REPORT

Dr. Rao V. S. Yalamanchili

NOVEMBER 1972

RESEARCH DIRECTORATE

WEAPONS LABORATORY USAWECOM

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

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A study is being made by personnel of the Research Directorate, Weapons Laboratory, WECOM to establish a capability to perform overall heat transfer analysis for any given dimensions of a weapon and for any specified propellant characteristics. With respect to this goal, the propellant gas convective heat transfer problem was divided into five subproblems: (1) generation of thermochemical properties for any given propellant; (2) transient inviscid compressible flow through the gun barrel, core flow; (3) transient viscous compressible flow on the bore surface, boundary layers; (4) unsteady heat diffusion through the gun tube, and (5) unsteady free convection and radiation outside the gun tube. An exact numerical solution for the unsteady, compressible, inviscid momentum and continuity equations by the method of characteristics was obtained. In general, the linear velocity gradient assumption was much more appropriate than the uniform density assumption. However, for the first portion of bullet travel (approximately 20 per cent), neither of these assumptions was valid. Analytical boundary layer analysis procedure was developed on the basis of the transient compressible boundary layer momentum integral equation. Convective heat transfer was evaluated with the Chilton-Colburn analogy. Input, output, and the computer program listing are given for the Lagrange approximation and for the exponential wall shear laws. The free convection and radiation around the gun tube are of the same order of magnitude. The coupling between the problems was also discussed.

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ABSTRACT

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CONTENTS

I INTRODUCTION

As the projectile in a weapon moves ahead because of the high-pressure gases created by the burning propellant, the propellant gas will be set into motion starting from rest Since the governing equations of fluid dynamics, for many problems of interest, are a system of nonlinear partial differential equations dominated by real gas and nonequilibrium effects, no general solutions exist in which arbitrary initial and boundary conditions are allowed Therefore, the flow field should be examined and the overall problem should be subdivided by a consideration of the dominant features only, i.e., (1) generation of thermochemical properties for hot gases, (2) transient inviscid compressible flow through the central section of the barrel, (3) unsteady compressible boundary layer flow near the bore surface, (4) transient heat diffusion through the multilayered gun barrel walls, and finally (5) unsteady free convection and radiation outside the gun tube.

Constant gas (specific heat and gas constant) and transport (thermal conductivity, viscosity) properties are common assumptions Similarly, in the case of problem (2), the use of uniform density, uniform temperature, and linear velocity gradient in a gas flow between the breech and the bullet is popular. Since the governing equations of this problem are of the hyperbolic type, they can be solved by the well-known method of characteristics Some of these results were presented at the 17th Army Mathematicians Conference, Redstone Arsenal, Ala None of these assumptions are considered reasonable until after peak heating occurs In general, however, the linear velocity gradient assumption is better than the uniform density assumption Problem (3) can also be interpreted as forced convective heat transfer in guns The magnitude of convective heat transfer in guns is large primarily because of the high values of gas densities that exist, and because of the large gas-to-wall temperature differences which constitute the convective heat transfer driving potential Correlation of experimental data is quite common with the various analytical empirical techniques The most popular method for heat transfer correlation purposes derives from the postulates of Nordheim, Soodak, and Nordheim [2] These authors theorize that the propellant gas wall shear friction factor is dependent only on gun surface roughness considerations The justification for elimination of the Reynolds number as a parameter is based on an order of magnitude estimates of boundary layer thickness with laminar boundary layer considerations The recommended form of the friction factor, thus, is dependent upon only the ratio of surface roughness to barrel radius Consequently, the friction factor is assumed to be independent of space or time within a given

gun barrel. Reynolds' analogy is subsequently utilized, and the heat transfer coefficient becomes simply proportional to gas density, velocity, and specific heat. Total calorimeter data have subsequently been rationalized in terms of the value of the friction factor which together with internal ballistic considerations obtains the spatial variation of heat load to the gun barrel derived from a single shot. Example values of experimental friction factor ($C_f/2 = \tau_w/\rho V^2$) reported vary from 0011 to .0035. Geidt[3] assumes a value of .002 and finds that his measured heat flux is predicted generally within about a factor of 2. Others have interpreted pressure gradients within the gun barrel in an attempt to evaluate wall shear for the application of Reynolds' analogy for heat transfer.

Another approach for correlation of experimental data is based on the Dittus-Boelter relation for steady, fully developed, turbulent pipe flow. This relation is also judged to be inaccurate in several references. Cornell Aeronautical Laboratory[4] proposed a combined analytical-experimental approach based on steady fully developed pipe flow concepts.

The state of the art in unsteady boundary layers and turbulent models is quite limited. Foster-Miller Associates[5] solved unsteady boundary layers with several approximations by the integral method and the method of characteristics. Aerotherm[6] under a contract with the Navy solved unsteady laminar boundary layer equations by the integral matrix procedure. Georgia Tech[7] under a contract with Eglin Air Force Base developed a solution procedure by the combination of integral methods and finite differences for turbulent boundary layers. Aerotherm[8] under a contract with the Research Directorate, Weapons Laboratory at Rock Island, developed an analytical boundary layer procedure based on the compressible time-dependent boundary layer momentum integral equation. Special solutions of the momentum equation were obtained by the classical Lagrange approximation and exponential wall shear laws. The solution was obtained by the method of characteristics. Convective heat transfer was evaluated based on the Chilton-Colburn analogy[9] of the energy boundary layer to the momentum boundary layer.

Since the momentum and energy boundary layer equations are not similar in an accelerating flow, an approximate solution of the energy integral equation is expected to yield better heat transfer results than applications of the Chilton-Colburn analogy to an approximate solution of the momentum integral equation. Another investigation is in progress to determine the validity of this analogy for unsteady boundary layers and to obtain a modified form if significant differences exist at least for a simple case such as unsteady incompressible boundary layers.

The unsteady free convection and radiation are discussed in Section VI. These two contributions are of the same order of magnitude. Since the surface conditions change quite rapidly for automatic weapons, the governing time-dependent partial differential equations are being solved by numerical methods. The estimations based on pure convection show that the flow around the gun tube is still in the laminar range.

The solution of transient heat diffusion through gun barrel walls was established quite satisfactorily by analytical[10], finite difference[11], and finite element[12,13,14] techniques. These are satisfactory for the purpose of establishing propellant gas convective heat transfer coefficients.

II. THERMOCHEMICAL PROPERTIES OF PROPELLANT GASES

As the propellant burns, more and more hot gases will be generated. Typical gas in a chamber contains a temperature of 5000°R and a pressure of 50,000 psi. The thermochemistry of propellants involves determination of chemical composition of gases (either by finite rate chemistry or by chemical equilibrium analysis) and the derivation of gas properties from the composition.

The ideal gas equation of state does not represent the gases that exist in a gun barrel. For interior ballistic purposes, the id al gas equation of state is commonly modified by a covolume correction. The covolume term is interpreted as the space occupied by the gas when compressed to the limit. This covolume term has the empirical value of approximately 1 cubic centimeter per gram for most of propellant gases. This is insignificant in solid propellant fueled rocket motors, but, it is quite significant in gun interior ballistic computations due to the high pressures involved.

Covolume correction alone does not represent the true nature of the gas for the entire ballistic cycle. Drastic changes in chemical composition, pressure, and temperature occurs throughout the firing cycle.

Thermochemical properties of typical propellant gases are being predicted by use of a NASA-LEWIS thermochemical program (CEC 71). The gases were highly toxic. (Typical composition of the gases: M18 ($CO = 0.47$, $H_2 = 0.19$, $H_2O = 0.16$, $N_2 = 0.1$, $CO_2 = 0.08$) and IMR ($CO = 0.43$, $H_2 = 0.12$, $H_2O = 0.21$, $N_2 = 0.12$, $CO_2 = 0.12$)). IMR is better than M18 as far as combustion is considered, however, it it still a long way from possible complete combustion. The consequences of incomplete combusti

are muzzle flash, smoke, and fire in addition to low efficiency. Small differences were observed between expansion of frozen and chemical equilibrium flows. However, quite significant differences exist for flame temperature, density, and force of propellant between the present results (Chemical Equilibrium Chemistry) and the values reported in Reference 15, which were obtained by shortcut calculational methods Therefore, future work should involve chemical equilibrium chemistry by consideration of all possible species and reactions (more than one hundred); and, if possible, finite rate chemistry should be included at a later date

For gases at moderately high pressure, the following equation has been recommended[16]:

$$\frac{PV}{RT} = 1 + \frac{B(T)}{V} + \frac{0.625\ b_0^2}{V^2} + \frac{0.2869\ b_0^3}{V^3} + \frac{0.1928\ b_0^4}{V^4}$$

where B(T) = Second virial coefficient

and b_0 = Vander Waals Constant

For powder gases at temperatures around 2500°C, the Second virial coefficient is almost independent of temperature A maximum value of B(T) (= .525 b_0) is present that corresponds to the high temperatures found in powder-gas flames The value of b_0 for the powder gas mixture may be estimated by the mixture rule:

$$(b_0)_{mix} = \sum_i X_i\ (b_0)_i$$

where X_i = mole fractions of individual components

and $(b_0)_i$ = Lennard-Jones b_0's for individual components.

III CLASSIFICATION OF FLOW IN GUN BARRELS

The transport properties of propellant gases are much more important that the thermodynamic properties as far as forced convection is concerned The thermodynamic properties and chemical composition of propellant gases were discussed in Section II The convective heat transfer coefficient is directly proportional to the thermal conductivity of gases. The thermal conductivity of propellant gases that are in use

could probably be increased 100 per cent. Since the mean free-path of gas molecules is inversely proportional to the pressure and the number of molecules per unit volume is directly proportional to the pressure, the thermal conductivity (of gases) that is a product of the two is not a strong function of pressure. However, the thermal conductivity is dependent strongly on the temperature of gases. Toward the goal of establishing correct values the measurement of thermal conductivity of gases at elevated temperatures is in progress under a contract (DAAF03-71-C-0320) with the University of Wisconsin

Flow in a laminar boundary layer will eventually become unstable as the Reynolds number is increased The boundary layer thickness, skin friction and heat transfer increases significantly more in turbulent flow than in laminar flow The eddy viscosity is the dominating mechanism for such increase The effect of eddy viscosity may be interpreted as if the gas has a viscosity several orders of magnitude greater than its molecular (kinematic) viscosity.

The flow characteristics are unknown for gun barrel flows. The experimental data are lacking because of the moving bullet. This obstacle may be overcome if one takes advantage of the similarities between the moving bullet (small mass) and a moving shock in a shock tube Therefore, shock tube data should be collected and analyzed for possible use in predicting gun barrel flow characteristics

As the propellant gases expand behind the projectile, a boundary layer forms at the breech end and thickens as the flow proceeds downstream. An unusual feature of the velocity boundary layer is that it disappears as the projectile is approached since all fluid at the base of the projectile must be moving at projectile velocity Mathematically, this amounts to the requirement of an additional boundary condition at a downstream location The numerical techniques applied to most boundary layer problems call for the specification of profiles at the upstream end of the flow a 'allow a marching" along the flow direction. For the usual time-dependent boundary layer problem, an initial condition to describe the boundary layer flow at time zero and boundary conditions as functions of time are required No downstream condition is added. The question logically arises then as to how boundary conditions at both ends of the flow can be satisfied in any one analysis Probably, the analyses may be carried out from both ends of the flow to match the solutions somewhere in the middle of flow, and thus the question posed above may be explained In the past, three separate criteria have been used to characterize fully developed turbulent flow in pipes and channels.

Fully turbulent flow has been said to exist: (1) when the skin-friction coefficient is related to the Reynolds' number by an established law known to hold at high Reynolds' numbers, (2) when the velocity distribution in the wall region follows the well known law of the wall, or (3) when the flow is continuously turbulent, i.e., no intermittence is present. Some of the results[21] indicate that these criteria yield different Reynolds' numbers.

TRANSITION TO A TURBULENT BOUNDARY LAYER

The laminar boundary layer becomes unstable if the Reynolds' number becomes sufficiently high; thus small disturbances will be amplified causing transition to a turbulent type of boundary layer.

For a flat plate with zero pressure gradient, the laminar boundary layer has been shown experimentally to be quite stable for length Reynolds' numbers Re_x up to about 80,000 and can persist to a Reynolds' number of several million if the free stream turbulence is very low and the surface is very smooth. For engineering calculations, unless other information is available, transition can generally be assumed to occur in the 200,000 to 500,000 range. These figures are only approximate and may be good for a smooth surface with a fair amount of free stream turbulence.

The length Reynolds' number Re_x is inconvenient for a transition criterion since it is based on a constant free stream velocity and may not be meaningful if it is allowed to vary with x such as the accelerating flow in a gun barrel. In such circumstances, a local parameter is preferred such as momentum thickness (δ) instead of x. If a critical Reynolds' number, Re_x, of about 360,000 is chosen as a transition criteria for a constant free stream velocity, the equivalent criterion for accelerating flow becomes $Re_\delta = 0.664$ $(600) = 398.4$.

Note that the Reynolds' number based on local distance does not possess any boundary layer characteristics, whereas the Reynolds' number based on momentun thickness does represent the important parameter of the boundary layer. Since transition to a turbulent boundary layer depends strongly on the growth of the boundary layer, consideration of Reynolds' number on the basis of momentum thickness is logical to establish transition criteria.

LAMINARIZATION OF A TURBULENT BOUNDARY LAYER IN GUNS

The boundary layer is commonly assumed as turbulent for higher Reynolds' number flows. Although the boundary layer may be initially turbulent, several investigations of heat transfer from heated air flowing through cooled axisymmetric nozzles revealed an apparent reduction of turbulent transport of heat along the nozzles, even though the throat or narrowed part Reynolds' numbers based on diameter were as large as 4×10^5 to 2×10^6. This reduction in heat transfer is believed to be associated with the effect of flow acceleration on turbulence. From the low-speed, essentially constant property, boundary layer measurements (References 17 and 18) and flow observations (Reference 19), a turbulent boundary layer had become laminarlike near the wall; this was presumably due to the loss of turbulent transport in the wall vicinity. This process referred to as laminarization, apparently occurs when values of a parameter

$$K = \frac{\nu_e}{u_e^2} \frac{du_e}{dx}$$

exceed about 2×10^{-6}.

The effect of flow acceleration on the velocity and temperature profiles in the nozzle have exhibited the following trends:

(1) The velocity profiles become relatively flat in the outer part of the layer and lie above the 1/7 power relation.

(2) A laminar boundary layer velocity profile fits the measured profile satisfactorily in the wall vicinity.

(3) The temperature profiles are still in relatively good agreement with the 1/7 power relation in the outer part of the layer.

(4) The thickness of the thermal layer is considerably larger than that of the velocity boundary layer.

EFFECT OF RIFLING

Separation of flow into regions of low total-temperature near the surface and high total-temperature out toward the free stream by a high-velocity gas flow over a flat plate is well known. Other rectilinear motions also demonstrate such a

separation of the flow into regions of low and high total temperatures. Such an energy separation is much more pronounced in vortex motion than in rectilinear flows.

The super imposition of a rotational velocity on the normal Poiseuille flow creates lower total temperatures in the core of the flow, the effect becomes larger with increased rotation. Extremely large ratios of circumferential to axial velocity could create very low total temperatures in the core regions. The axial component of velocity prevents this separation from ever reaching the value obtainable with pure rotation. A greater separation effect exists in turbulent rotational flow than in laminar rotational flow.

IV. UNSTEADY CORE-FLOW ANALYSIS

The behavior of propellant systems is predicted almost entirely on the basis of intuition and experience, and the use of interior ballistic equations. With these equations space-mean averages are assumed for the thermodynamic variables and include dynamic effects only through the inclusion of various correction factors determined more or less, empirically.

In the literature, either uniform density or a linear velocity gradient is assumed to be in the flow field between the breech and the bullet at any particular time during weapon firing The objective of this study is to evaluate the validity of these assumptions in interior ballistic analyses.

The governing equations, i.e., continuity, momentum, and energy of unsteady, inviscid, and compressible flow are hyperbolic in nature. In principle, one can solve them by the well-known method of characteristics. Along certain (characteristic) directions, the coupled partial differential equations can be reduced to a set of simultaneous ordinary differential (characteristic) equations. The solution of such a set of equations is further complicated by an unknown nonhomogeneous, and unsteady moving (bullet) boundary condition and a time-dependent propellant burning model.

The external or wall friction and propellant burning models are considered in the formulation of the governing equations. The Nobel-Abel equation of state is used to represent the state of the gas. The governing equations are nondimensionalized; equations are derived for characteristic lines and also for dependent variables along those characteristic directions.

As the bullet moves ahead because of the high-pressure gases, it issues rarefaction waves which will travel toward the breech The reflected waves from the breech will interact with the oncoming rarefaction waves and thus form a network similar to a coordinate grid system. The path of these waves may be called characteristic lines. The applicable ordinary differential equations along these lines can be solved by finite difference techniques.

The core flow model, while necessarily a gross oversimplification of the physical system, would permit at least a study of the distribution of thermodynamic variables and provide more accurate interior ballistics data than the data obtained by conventional methods to aid in the design of muzzle brakes, noise and flash suppressors, and also gas ports for automatic actuating mechanisms

MATHEMATICAL MODEL

A model involving unsteady, one-dimensional motion would apparently be the most that could be tolerated in complexity. Moreover, this model does not warrant any more complexity due to the limited knowledge in solid propellant particle movements and in associated burning models The gas is assumed to be inert and also without viscosity and heat conduction effects. For convenience, the mass and energy are assumed to be distributed continuously throughout the gas flow, although such an assumption is unnecessary to apply the chosen method of approach for the solution to the problem. The governing equations are given below

Continuity.

$$\frac{\partial \rho}{\partial \tau} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \rho u \frac{\partial}{\partial x} \ln A = Q\left(1 - \frac{\rho}{\delta}\right) \tag{1}$$

Momentum:

$$\frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{u}{\rho} Q - \frac{2f}{D} u^2 \tag{2}$$

Energy:

$$\rho \frac{\partial}{\partial \tau}\left(e + \frac{u^2}{2}\right) + \rho u \frac{\partial}{\partial x}\left(e + \frac{u^2}{2}\right) + Q\left(e + \frac{u^2}{2}\right) = WQ - \frac{1}{A} \frac{\partial}{\partial x} (pAu) \quad (3)$$

Equation of State.

$$p\left(\frac{1}{\rho} - \eta\right) = RT \quad (4)$$

where

ρ = gas density

u = velocity

A = Area

Q = mass of gas produced per unit volume per second

δ = propellant density

p = pressure

f = friction factor

e = internal energy

W = potential of propellant

η = covolume of propellant gas

R = gas constant

The wall friction does not appear directly in the energy equation since its action is that of converting mean kinetic energy to thermal energy

THERMODYNAMIC RELATIONS

Entropy.

$$de = C_V \, dT = T \, dS + \frac{p}{\rho^2} d\rho \quad (5)$$

or

$$\frac{ds}{C_V} = \frac{dp}{p} - \frac{C_V + R}{C_V} \frac{d\rho}{\rho^2\left(\frac{1}{\rho} - \eta\right)} \quad (6)$$

$$C_p - C_V = T\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_V \tag{7}$$

For Nobel-Abel gas, $\left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p}$; $\left(\frac{\partial p}{\partial T}\right)_V = \frac{R}{\frac{1}{\rho} - \eta}$

$$\therefore C_p - C_V = R \text{ for Nobel-Abel gas} \tag{8}$$

Substituting Equation 8 into Equation 6 and integrating the resulting equation yields the following relationship:

$$p\left(\frac{1}{\rho} - \eta\right)^{\gamma} = C\, e^{S/C_V} \tag{9}$$

where

C_V = specific heat of gas at constant volume

γ = specific heat ratio

S = entropy

C = integration constant

<u>Speed of Sound</u>:

The square of the speed of sound is defined as

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_S \tag{10}$$

The following equation can be obtained by use of Equations 9 and 10:

$$a^2 = \frac{\gamma p}{\rho^2\left(\frac{1}{\rho} - \eta\right)} \tag{11}$$

Equation 11 can also be expressed in differential form as

$$2\,\frac{da}{a} = \frac{dp}{p} - \frac{1 - 2\rho\eta}{1 - \rho\eta}\,\frac{d\rho}{\rho} \tag{12}$$

Transformation of Governing Equations:

The governing equations can be conveniently written with u, a, and S instead of u, ρ, and e as dependent variables. Toward this goal, the following equations are derived by use of Equations 6, 8, and 12.

$$\frac{d\rho}{\rho} = \frac{1 - \rho\eta}{\gamma - 1 + 2\rho\eta}\left[2\frac{da}{a} - \frac{dS}{C_V}\right] \tag{13}$$

and

$$\frac{dp}{p} = \frac{2\gamma}{\gamma - 1 + 2\rho\eta}\frac{da}{a} - \frac{1 - 2\rho\eta}{\gamma - 1 + 2\rho\eta}\frac{dS}{C_V} \tag{14}$$

Equation 1 can be transformed into the following form by use of Equation 13.

$$\frac{\partial a}{\partial \tau} + u\frac{\partial a}{\partial x} + \frac{\gamma - 1 + 2\rho\eta}{1 - \rho\eta}\cdot\frac{a}{2}\cdot\frac{\partial u}{\partial x} - \frac{a}{2C_V}\left(\frac{\partial S}{\partial \tau} + u\frac{\partial S}{\partial x}\right) = \frac{\gamma - 1 + 2\rho\eta}{2(1 - \rho\eta)}\, a\, Q\left(\frac{1}{\rho} - \frac{1}{\delta}\right) \tag{15}$$

Similarly, the momentum equation can be reduced into the following form by use of Equation 14:

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + \frac{2a(1 - \rho\eta)}{\gamma - 1 + 2\rho\eta}\frac{\partial a}{\partial x} - \frac{a^2(1 - \rho\eta)(1 - 2\rho\eta)}{C_p(\gamma - 1 + 2\rho\eta)}\frac{\partial S}{\partial x} = -\frac{2fu^2}{D} - \frac{uQ}{\rho} \tag{16}$$

The transformed energy equation is given below:

$$\frac{\partial S}{\partial \tau} + u\frac{\partial S}{\partial x} = \frac{Q}{\rho T}\left[W + \frac{u^2}{2} - e - p\left(\frac{1}{\rho} - \eta\right)\right] \tag{17}$$

This is obtained by the combination of Equations 1, 2, 3, 6, and 8.

Characteristic Directions:

The characteristic directions are defined as the curves along which the derivatives of the fluid properties such as

$\frac{\partial u}{\partial \tau}, \frac{\partial u}{\partial x}, \frac{\partial a}{\partial \tau}, \frac{\partial a}{\partial x}, \frac{\partial S}{\partial \tau}, \frac{\partial S}{\partial x}$ are discontinuous. The equations for

these curves can be determined by consideration of the following three definitions along with Equations 15, 16, and 17 as a system of linear algebraic equations with variables

$\frac{\partial u}{\partial \tau}, \frac{\partial u}{\partial x}, \frac{\partial a}{\partial \tau}, \frac{\partial a}{\partial x}, \frac{\partial S}{\partial \tau}$ and $\frac{\partial S}{\partial x}$

$$\frac{\partial u}{\partial \tau}d\tau + \frac{\partial u}{\partial x}dx = du \tag{18}$$

$$\frac{\partial a}{\partial \tau}d\tau + \frac{\partial a}{\partial x}dx = da \tag{19}$$

$$\frac{\partial S}{\partial \tau}d\tau + \frac{\partial S}{\partial x}dx = dS \tag{20}$$

One can obtain Equation 21 for $\frac{\partial u}{\partial \tau}$ by solving the above system of equations simultaneously (See page 14)

Setting the denominator to zero and performing algebraic manipulations yields the following equations for characterisitc directions

$$\frac{dx}{d\tau} = u + a \tag{22}$$

$$\frac{dx}{d\tau} = u - a \tag{23}$$

$$\frac{dx}{d\tau} = u \tag{24}$$

Governing Equations Along Characteristics Directions:

The governing partial differential equations, namely Equations 15, 16, and 17, can be reduced to simple ordinary differential equations along the characteristic directions. The reduction can be done by setting the numerator of Equation 21 to zero and performing required algebraic manipulations This procedure explicitly uses the definition of characteristic direction Thus, the simplified governing equations along the characteristic directions (Equation 22, 23, and 24, respectively) will become equations 25, 26, and 27.

Solving for $\frac{\partial u}{\partial \tau}$, one can obtain in determinant notation:

$$\frac{\partial u}{\partial \tau} = \frac{\begin{vmatrix} \frac{(\gamma - 1 + 2\rho\eta)aQ}{2(1 - \rho\eta)}(\frac{1}{\rho} - \frac{1}{\delta}) & (\frac{\gamma - 1 + 2\rho\eta}{1 - \rho\eta})\frac{a}{2} & 1 & u & \frac{-a}{2C_V} & \frac{-au}{2C_V} \\ -\frac{uQ}{\rho} - \frac{2f}{D}u & u & 0 & \frac{2a(1 - \rho\eta)}{\gamma - 1 + 2\rho\eta} & 0 & \frac{-a^2(1 - \rho\eta)(1 - 2\rho\eta)}{C_p(\gamma - 1 + 2\rho\eta)} \\ \frac{Q}{\rho T}[W + \frac{u^2}{2} - e - p(\frac{1}{\rho} - \eta)] & 0 & 0 & 0 & 1 & u \\ du & dx & 0 & 0 & 0 & 0 \\ da & 0 & d\tau & dx & 0 & 0 \\ ds & 0 & 0 & 0 & d\tau & dx \end{vmatrix}}{\begin{vmatrix} 0 & (\frac{\gamma - 1 + 2\rho\eta}{1 - \rho\eta})\frac{a}{2} & 1 & u & \frac{-a}{2C_V} & \frac{-au}{2C_V} \\ 1 & u & 0 & \frac{2a(1 - \rho\eta)}{\gamma - 1 + 2\rho\eta} & 0 & \frac{-a^2(1 - \rho\eta)(1 - 2\rho\eta)}{C_p(\gamma - 1 + 2\rho\eta)} \\ 0 & 0 & 0 & 0 & 1 & u \\ d\tau & dx & 0 & 0 & 0 & 0 \\ 0 & 0 & d\tau & dx & 0 & 0 \\ 0 & 0 & 0 & 0 & d\tau & dx \end{vmatrix}} \tag{21}$$

$$\frac{Du}{D\tau} + \frac{2(1-\rho\eta)}{\gamma - 1 + 2\rho\eta}\frac{Da}{D\tau} = \frac{a}{C_p}\frac{(1-\rho\eta)(1-2\rho\eta)}{\gamma - 1 + 2\rho\eta}\frac{DS}{D\tau}$$

$$+ \frac{aQ}{C_p}(1-\rho\eta)(W + \frac{u^2}{2} - C_pT) + aQ(\frac{1}{\rho} - \frac{1}{\delta}) - \frac{uQ}{\rho} - \frac{2f}{D}u^2 \qquad (25)$$

$$\frac{Du}{D\tau} - \frac{2(1-\rho\eta)}{\gamma - 1 + 2\rho\eta}\frac{Da}{D\tau} = -\frac{a}{C_p}\frac{(1-\rho\eta)(1-2\rho\eta)}{\gamma - 1 + 2\rho\eta}\frac{DS}{D\tau}$$

$$- \frac{aQ}{C_p}(1-\rho\eta)(W + \frac{u^2}{2} - C_pT) - aQ(\frac{1}{\rho} - \frac{1}{\delta}) - \frac{uQ}{\rho} - \frac{2f}{D}u^2 \qquad (26)$$

$$\frac{DS}{D\tau} = \frac{Q}{\rho T}[W + \frac{u^2}{2} - e - p(\frac{1}{\rho} - \eta)] \qquad (27)$$

The dependent and independent variables are nondimensionalized with appropriate reference values. The new variables are as follows:

$\bar{u} = \frac{u}{a_o}$, $\bar{a} = \frac{a}{a_o}$, $\bar{x} = \frac{x}{L}$, $\bar{\tau} = \frac{a_o\tau}{L}$ and $\bar{S} = \frac{S}{C_p}$ where L is the length of the barrel.

Propellant Burning Model:

The propellant burning model is formulated by the assumption that the propellant charge conforms to the most common burning rate equation in interior ballistics, i.e.,

$$r = kp^n \qquad (28)$$

This is an empirical equation primarily based on closed vessel tests such as the bomb calorimeter. The terms k and n are the ballistic properties and are available for almost all of the solid propellants that are in use today.

The flow rate of the burnt gas into the system can be obtained from Equation 28 by multiplication of it by the propellant density, δ and the burning surface area, σ.

I.e.,

$$Q = \frac{\delta r \sigma}{\omega} \tag{29}$$

Where

$$\omega = \sigma \int_0^{\tau} r\,dt \tag{30}$$

Boundary Conditions:

The gases are at rest at the breech. The gases at the base of the projectile are allowed to move with the projectile velocity because of a continuity requirement.

As the projectile moves ahead because of the high pressure gases created by the burning propellant, the propellant gas will be set into motion. At every instant of the motion, a rarefaction wave originates at the bottom of the projectile and travels backward through the compressed gas and causes the gas to expand. However, from among these infintely many wavelets only a few are selected, to carry on the solution procedure.

If the stepwise projectile curve undergoes a change in slope at a point where no incident wave occurs, the boundary conditions require that a new wave be propagated into the gas from the point where the projectile changes in velocity.

The reflected wave from a moving object may be either a rarefaction, a compression or of zero strength, depending upon whether the new object speed is greater than, less than, or equal to the fluid velocity at the previous object speed. Therefore, the reflected wave will always be a rarefaction wave in the case of a bullet.

Even though continuity is required between a bullet and the gas at the base of the projectile, this is still an unknown boundary condition. However, a relation can be formulated by consideration of a dynamic force balance for the moving projectile,

i.e.,

$$m\frac{dv}{d\tau} = pA - F \tag{31}$$

where m = mass of the projectile
v = velocity of the projectile
and F = friction between projectile and the bore

Numerical Solutions:

The interaction of rarefaction waves originated from the base of the projectile, and the reflected waves from the breech

are shown in Figure 1. This is constructed by the use of Equations 22 and 23. In doing so, the curved characteristics between the nodes are replaced by straight lines. This procedure also yields the projectile path as a function of time, τ. To accomplish this requires an iteration scheme because of coupling between the governing equations and the moving (unknown) bullet boundary condition.

Two numerical examples are generated, one of which is applicable to small arms and the other to an artillery weapon. These are obtained on the assumption that the propellant is burnt completely before the projectile starts moving. The objective of introducing such a simplification is to obtain the distribution of fluid properties along the barrel at any time with minimum effort. This assumption is unnecessary for the solution procedure.

Only five rarefaction waves are initiated before the reflected wave from the breech intersects with the base of the projectile. The number of initiated waves is not a strong function of the net results; however, neither too many nor too few are desirable. A trial procedure is needed to obtain more accurate results.

Sufficient fluid property distribution throughout the barrel at any time cannot be obtained without some interpolation scheme. Linear variations for velocities and polytropic relationships for pressure, density, and temperatures are used for interpolation between the two neighboring nodes.

The dimensionless time versus the dimensionless pressure is plotted in Figure 2. This solution is also compared with an approximate solution obtained in Reference 20. The differences are believed to be primarily due to the assumption of uniform pressure throughout the barrel and the use of the perfect gas law. The bullet velocity versus distance along the barrel is shown in Figure 3. The lower velocities are obtained as anticipated due to the prediction of lower bullet base pressures than polytropic (approximate solution) pressures. Similar differences are noticed in Figure 4 between the method of characteristics solution and the gun heat transfer solution.[8,6]

The following data on small arms were used to obtain Figures 5, 6, and 7:

Barrel diameter = 1.625 in

Chamber length = 3.0 in

Charge mass = 0.05 lb

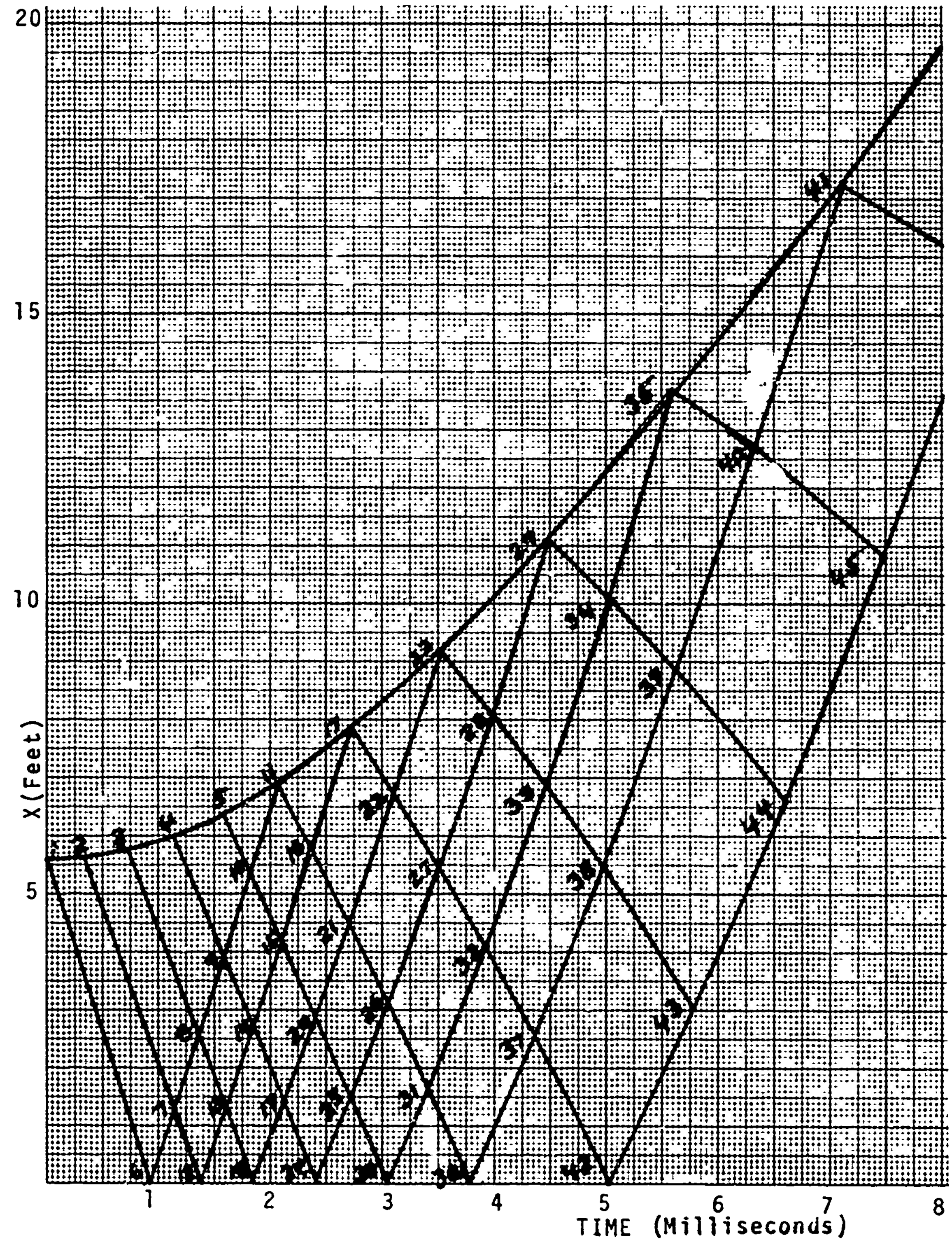

FIGURE 1 CHARACTERISTIC LINES (RAREFACTION WAVES)

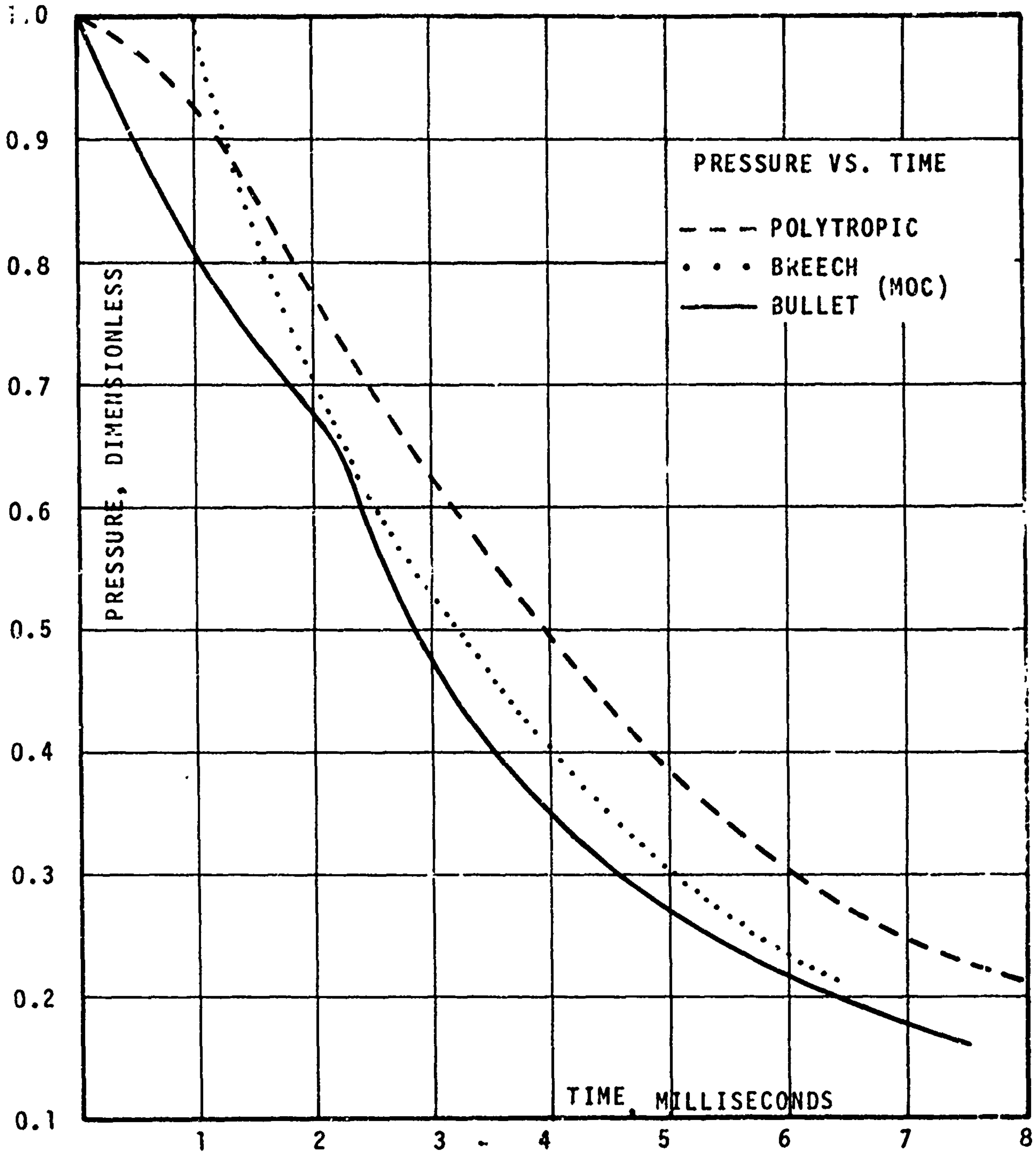

FIGURE 2 PRESSURE VERSUS TIME

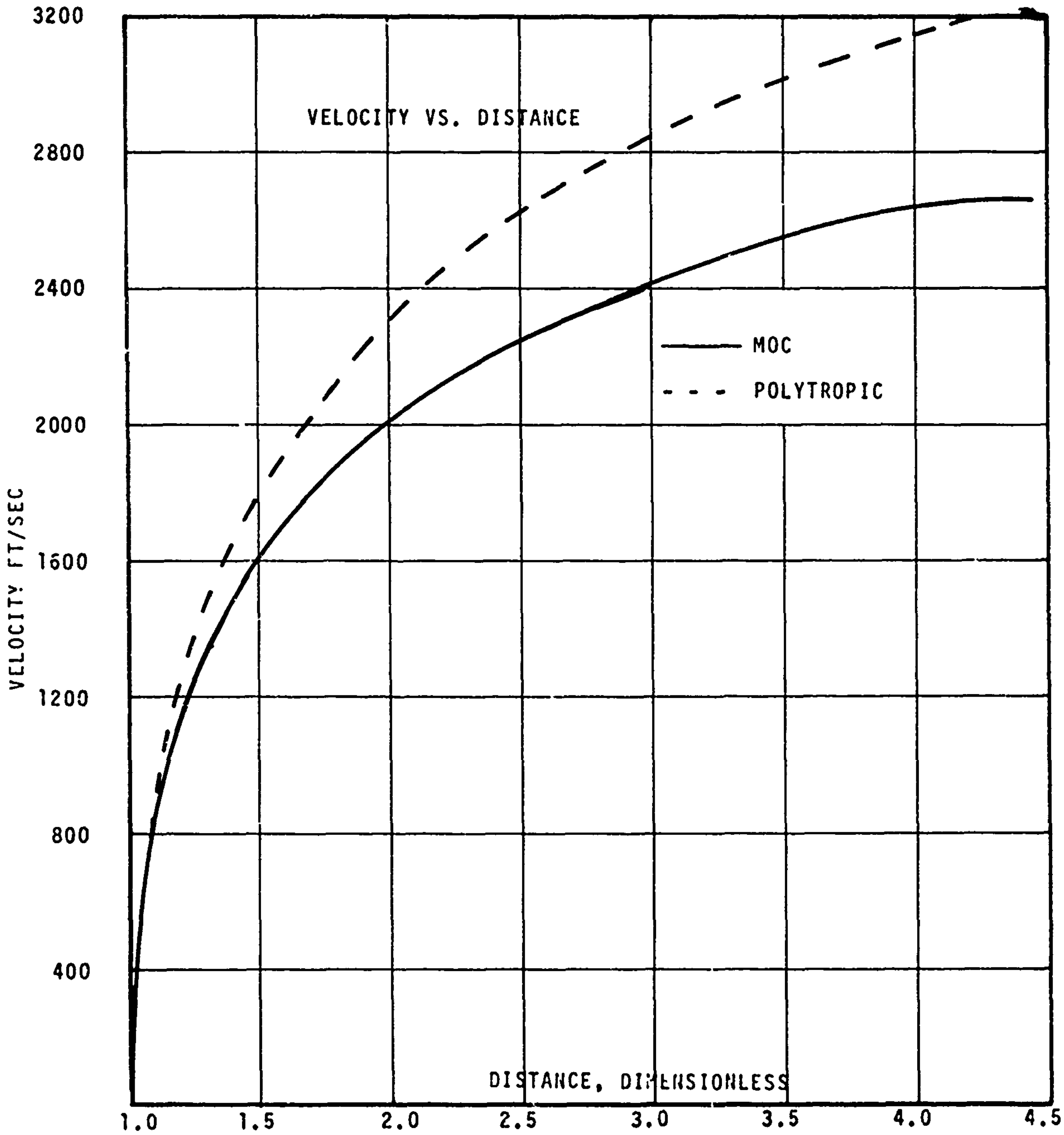

FIGURE 3 VARIATION OF PROJECTILE VELOCITY ALONG THE BARREL

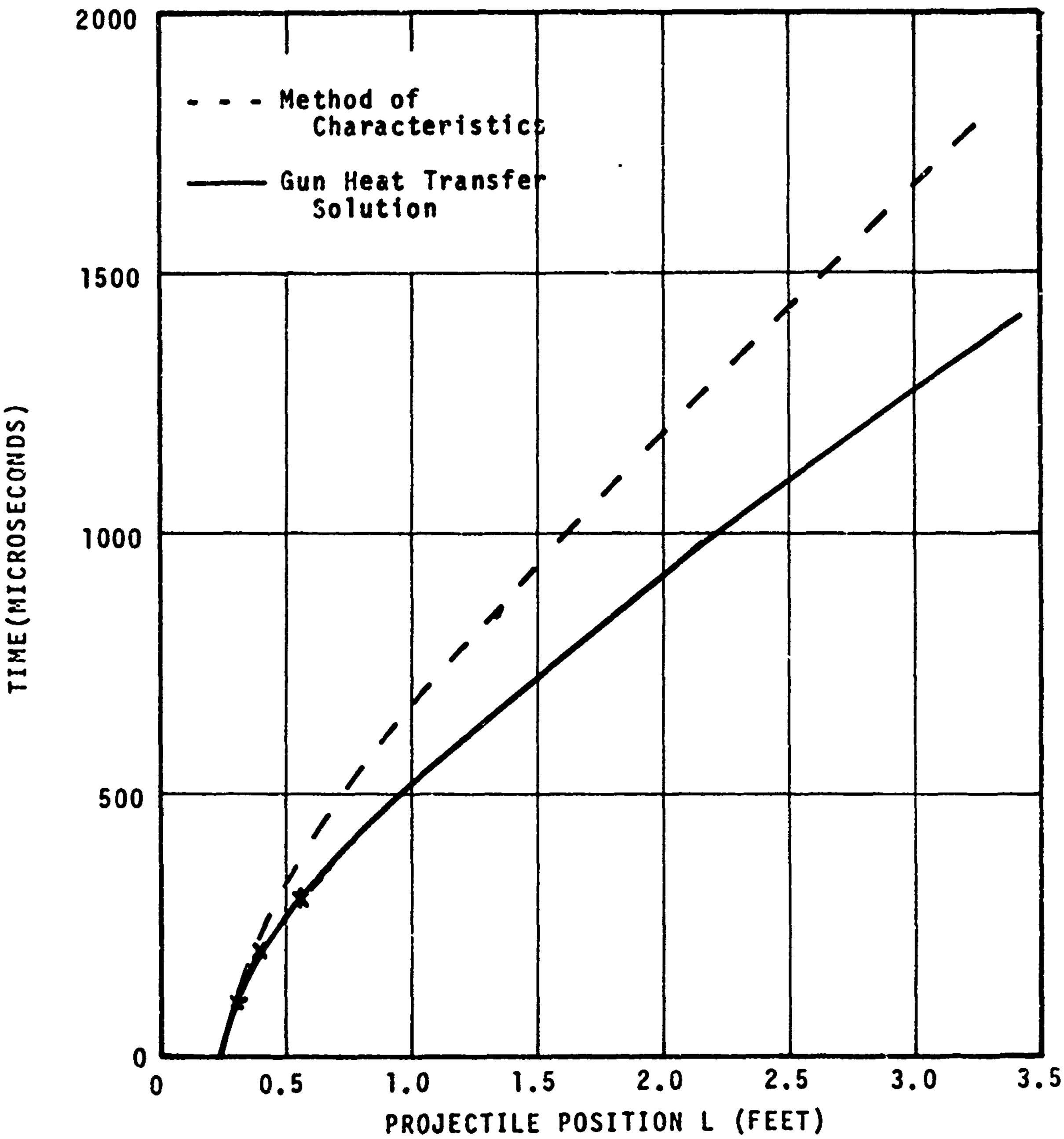

FIGURE 4 PROJECTILE POSITION VERSUS TIME

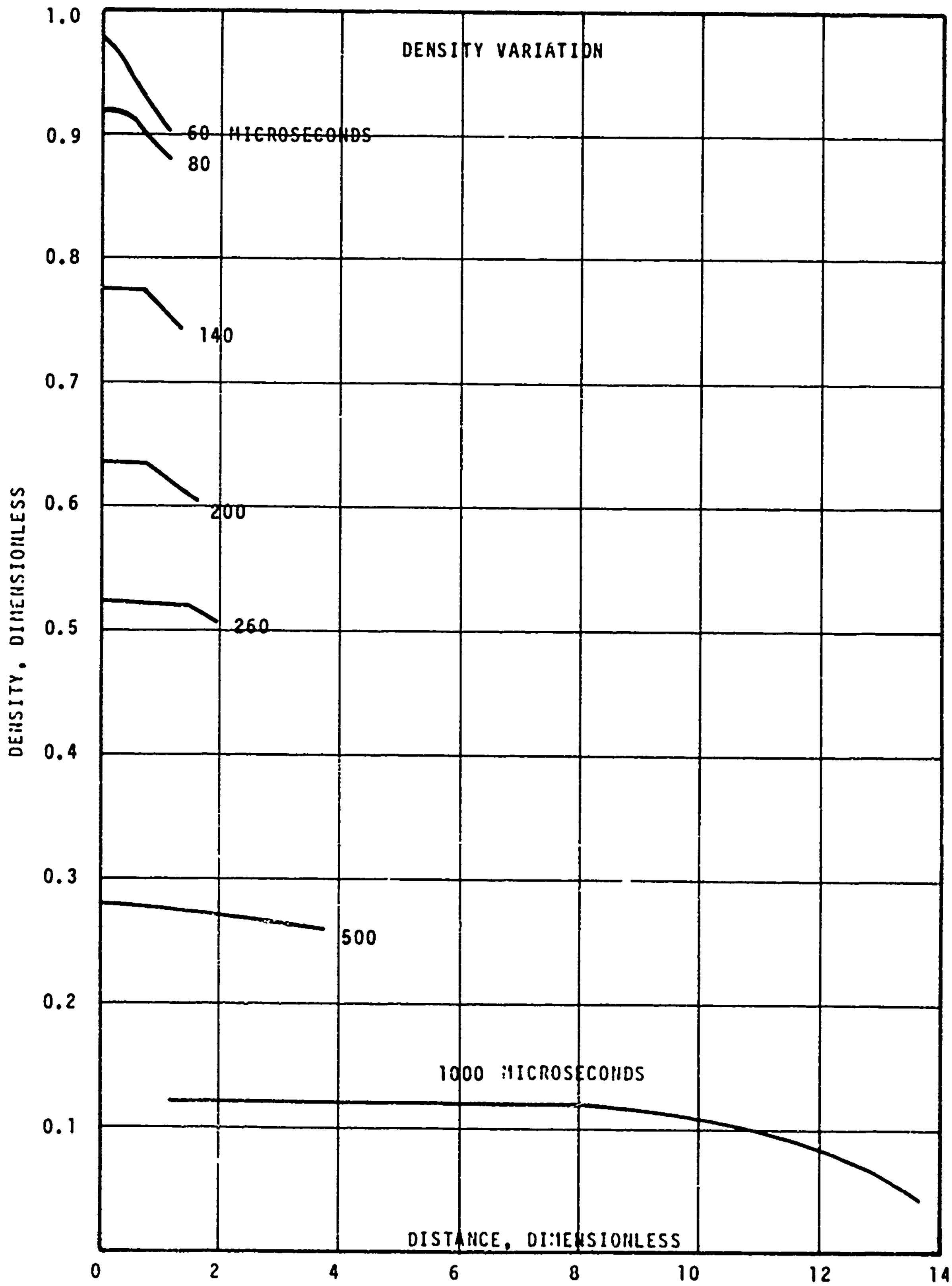

FIGURE 5 DENSITY DISTRIBUTION IN SMALL ARMS

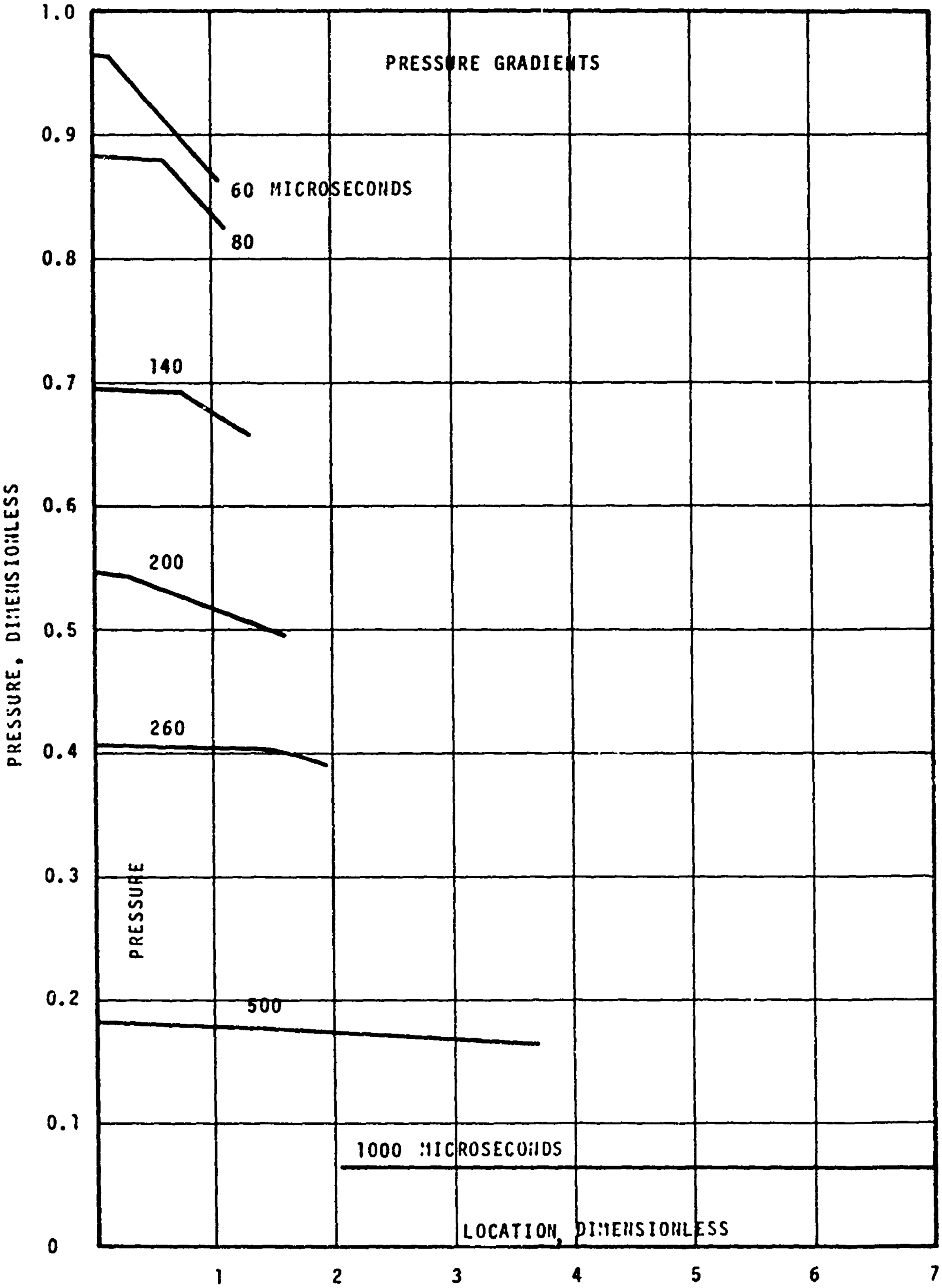

FIGURE 6 PRESSURE DISTRIBUTION IN SMALL ARMS

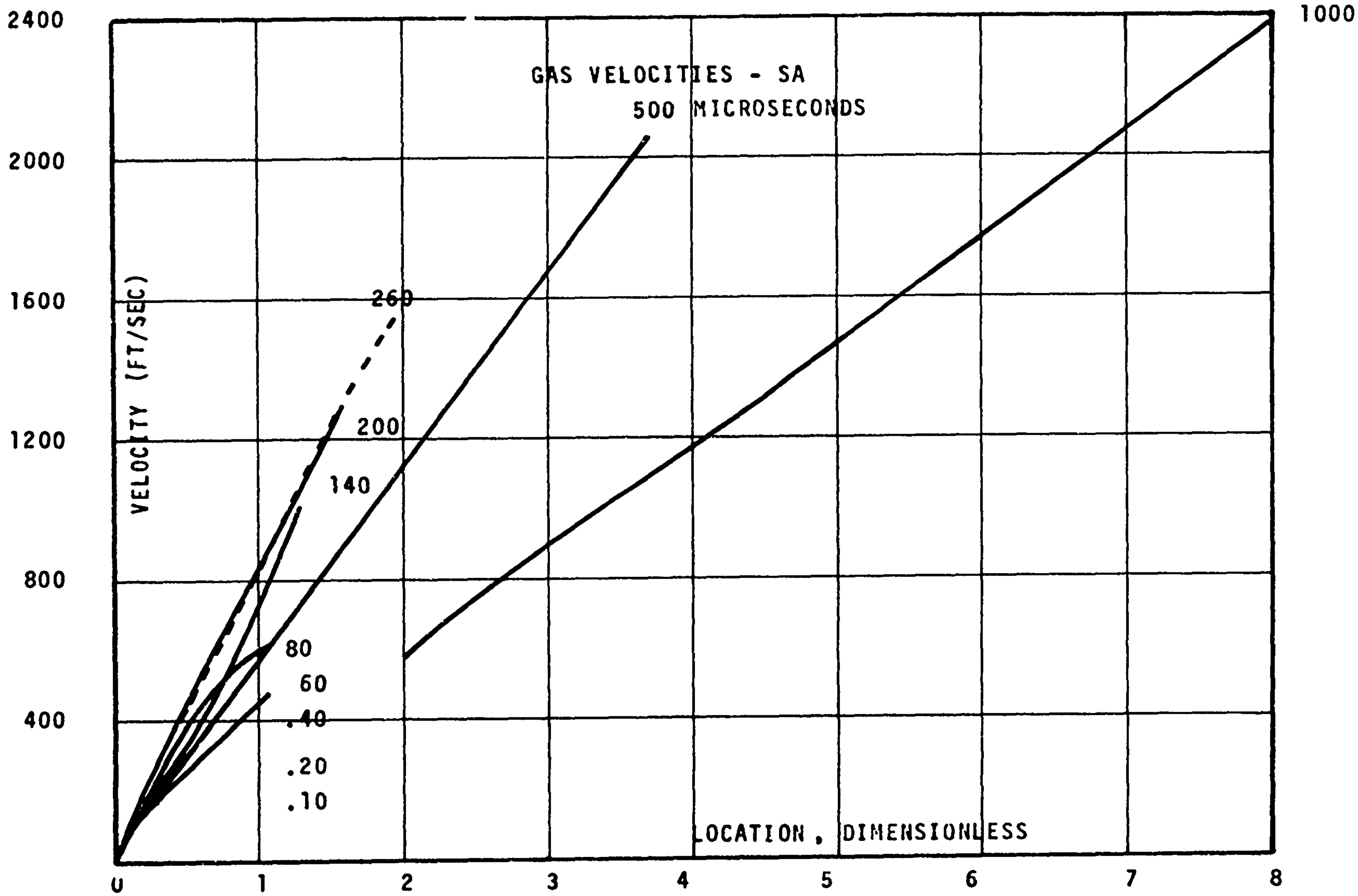

FIGURE 7 GAS VELOCITY DISTRIBUTION IN SMALL ARMS

Projectile mass	=	0.3 lb
Barrel length	=	41 in
Specific heat ratio	=	1.2
Adiabatic flame temperature	=	5200°F
Maximum pressure	=	33740 psia
Molecular weight	=	20

Obviously, no empirical relation can be devised for pressures and density, at least until the bullet leaves the muzzle. The data on those curves represent time in microseconds. Gas velocity distributions may be approximated as linear after about 20 per cent of the bullet travel. Similar results on the following weapon are shown in Figures 8 and 9:

Barrel diameter	=	0.493 ft
Barrel length	=	24.97076 ft
Chamber length	=	5.56 ft
Mass of the bullet	=	110.22 lb
Maximum pressure	=	13406329.4 psf
Maximum density	=	25 lb/ft^3
Covolume	=	0.016 ft^3/lb
Specific heat ratio	=	1 22222

In general, the assumption of a linear velocity gradient is better than the assumption of uniform density. Note that the uniform density assumption implies a linear velocity gradient due to the continuity equation. For the first portion of bullet travel (approximately 20 per cent), neither of these assumptions are good. For the remaining portion of bullet travel, the linear velocity gradient assumption is much better than the uniform density assumption. Further work is in progress to generalize these conclusions by the inclusion of variable propellant burning models. In this case, the validity of these assumptions is anticipated to be more questionable than in the previous case.

V. UNSTEADY BOUNDARY LAYER ANALYSIS

The rate of heat transfer from the hot propellant gases to the barrel is controlled by the development of the boundary layers. The flow in the gun barrel boundary layers could be laminar, transitional, or turbulent in nature. The type of boundary layer at a particular cross section at any instant need not be the same as at another instant. Since the flow

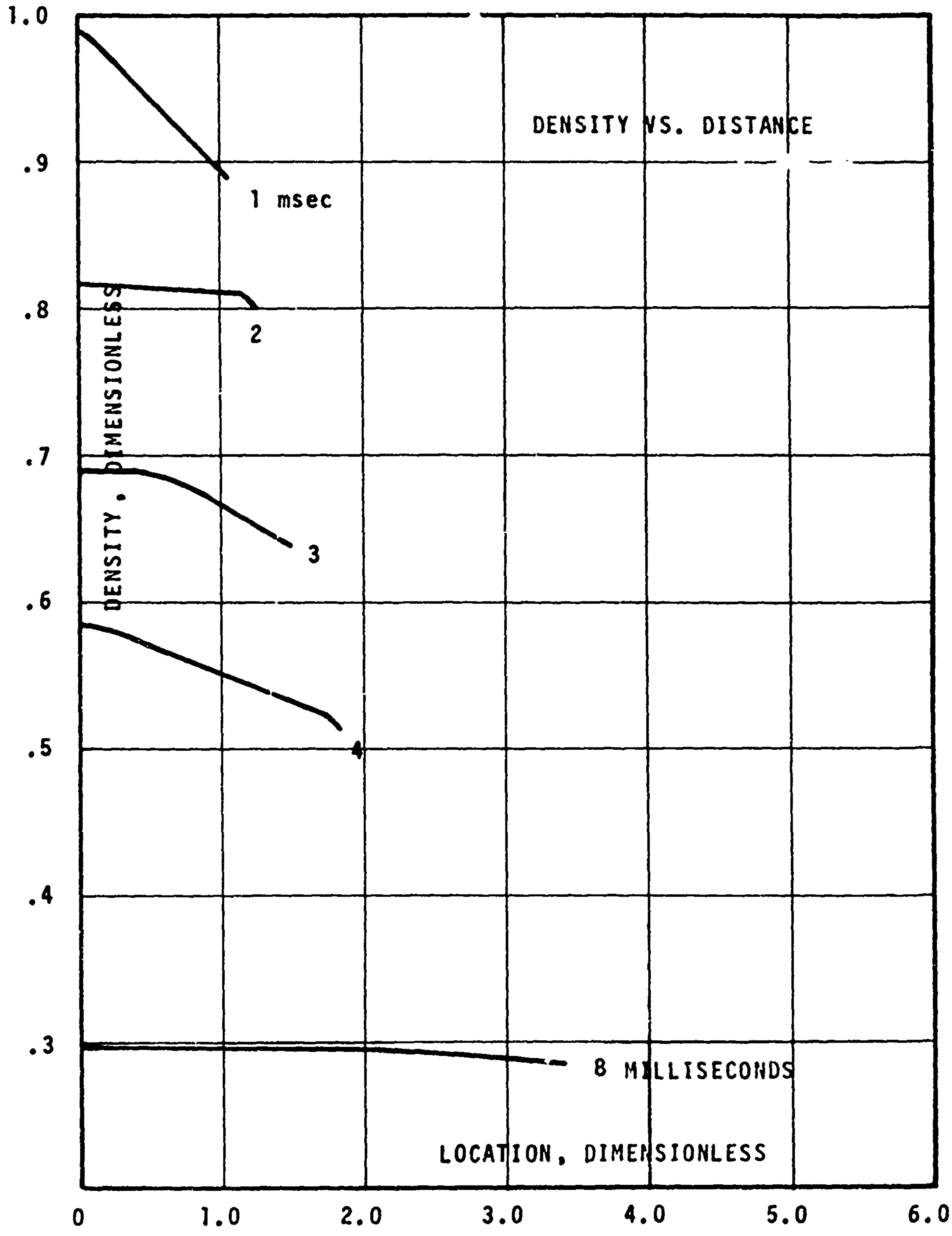

FIGURE 8 DENSITY DISTRIBUTION IN LARGE CALIBER WEAPONS

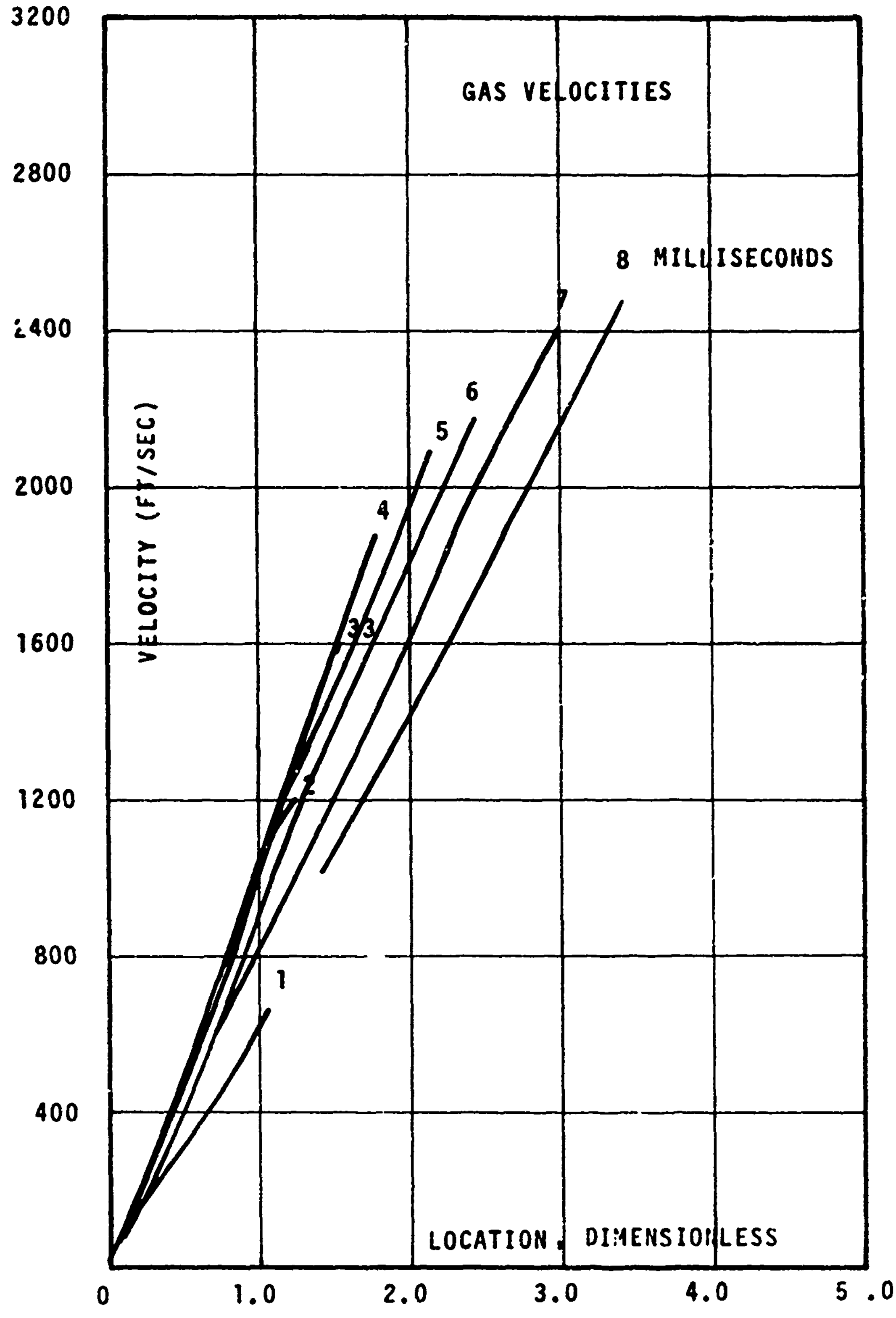

FIGURE 9 GAS VELOCITY DISTRIBUTION IN LARGE CALIBER WEAPONS

has to start from rest and must also satisfy zero boundary layer thickness at the bullet base because of the scraping action of bullet, laminar flow always exists in some parts of the gun barrel boundary layers.

Flow in a laminar boundary layer will eventually become unstable as the Reynolds' number is increased. The boundary layer thickness, skin friction, and heat transfer increases more rapidly in turbulent flow than in laminar flow The eddy viscosity is the dominating mechanism for such increases. The boundary layer flow can be turbulent somewhere in the middle of the flow between the breech and the bullet base A transitional regime should exist between the laminar and turbulent regimes. However, because of limited knowledge about transitional regimes, the flow will be assumed to change suddenly from laminar to turbulent flow at a time and place determined by a well-known laminar-turbulent transition criteria. Therefore, the unsteady boundary layer analysis is needed for laminar and turbulent boundary layers.

Toward this goal, a preliminary study was initiated with Aerotherm Corporation, Mountain View, California. The complete details of the analytical procedure developed under this contract can be found in Reference 8. The analytical procedure basically consists of an integral technique to remove the dependency of the transverse independent variable and the use of the method of characteristics to solve the resulting equations. Simplifications such as the Lagrange approximation, exponential wall shear laws, constant wall temperature, uniform density, pressure and temperature along the barrel, were introduced to reduce the complexity of computations.

The energy equation of unsteady boundary layers was not considered. Instead, the Chilton-Colburn analogy of relating the momentum transfer to the energy transfer was used to obtain the convective heat transfer coefficient. The important equations of the analytical procedure given above are summarized here for convenience.

Integral Momentum Equation.

$$\frac{\partial\theta}{\partial\tau} + A_1 \frac{\partial\theta}{\partial x} + (A_2 + A_3)\theta - A_4 = 0$$

$$A_1 = \frac{u_1}{G}$$

$$A_2 = A_1\frac{\partial}{\partial x}(\ln\rho_1) + \frac{2 + H}{G}\frac{\partial u_1}{\partial x}$$

$$A_3 = \frac{\partial}{\partial \tau}(\ln G) + \frac{H}{G}\frac{\partial}{\partial \tau}(\ln u_1) + \frac{\partial}{\partial \tau}(\ln\rho_1)$$

$$A_4 = \frac{u_1}{G}(\frac{C_f}{2} + B)$$

$$\frac{C_f}{2} = \frac{\tau_w}{\rho_1 u_1{}^2}$$

$$H = \frac{\delta^*}{\theta}$$

$$G = \frac{\delta^* - \delta^*_\rho}{\theta}$$

$$B = \frac{\rho_w V_w}{\delta_1 u_1}$$

$$\delta^*_\rho = \int_0^\delta (1 - \overline{\rho})dy$$

$$\delta^* = \int_0^\delta (1 - \overline{\rho}\,\overline{u})dy$$

$$\theta = \int_0^\delta \overline{\rho}\,\overline{u}\,(1 - \overline{u})dy$$

$$\overline{\rho} = \frac{\rho}{\rho_1}$$

$$\overline{u} = \frac{u}{u_1}$$

δ = boundary layer thickness

If the coefficients A_1 through A_4 can be expressed as functions of θ, x and t, then a single first-order equation for θ is obtained. To accomplish this, C_f in the A_4 term was related to θ by use of the following turbulence model:

$$\frac{u}{u_1\sqrt{\frac{C_f}{2}}} = \frac{y\,u_1}{\nu}\sqrt{\frac{C_f}{2}}, \quad 0 < \frac{y\,u_1}{\nu}\sqrt{\frac{C_f}{2}} < y_L$$

$$\frac{u}{u_1\sqrt{\frac{C_f}{2}}} = 5.8 + 5.2\log\left(\frac{y\,u_1}{\nu}\sqrt{\frac{C_f}{2}}\right), \quad y > y_L$$

y_L = dimensionless laminar sublayer thickness.

The results from this model were approximated by the following relation which also applies to laminar or turbulent conditions.

$$\frac{C_f}{2} = \frac{a^{\frac{1}{1-m}}}{\left[(1-m)\frac{u_1\theta}{\nu}\right]^{\frac{m}{1-m}}}$$

Reference properties were introduced to incorporate compressibility. The shape factors G and H were evaluated based upon the following assumptions:

$$\frac{u}{u_1} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

$$\frac{T}{T_1} = \frac{T_w}{T_1} + \left(1 - \frac{T_w}{T_1}\right)\frac{u}{u_1}$$

$$p\left(\frac{1}{\rho} - \eta\right) = RT$$

The quantities m and n can be related by the following relation.

$$n = \frac{2 - 3m}{m} \text{ or } m = \frac{2}{n + 3}$$

The specific value of G or H then depends entirely on T_w/T_1 ratio. The integral momentum equation along a characteristic curve

$$\frac{dx}{dt} = A_1$$

can further be reduced to the following ordinary differential equation:

$$\frac{D\theta}{Dt} + (A_2 + A_3)\theta - A_4 = 0$$

where D/Dt represents substantial or material derivative.

The two equations may be solved simultaneously by ordinary methods, if the variations of u_1 and ρ, along the barrel and with time are known. However, the following assumptions, in addition to the several mentioned above, permit analytical integration of those two ordinary differential equations:

Longitudinal gradients of gas and wall temperatures were assumed equal to zero

Longitudinal gradient of gas density was assumed equal to zero. This also implies a linear velocity gradient along the barrel (Lagrange approximation)

The results of the analytical integration are as follows:

Location of characteristic lines:

$$x = x_c e^{-f_2(t_c)} e^{f_2(t)}$$

Momentum thickness:

$$\theta = \frac{a}{1 - m}[x_c e^{-f(t_c)}]^{1 - 2m} \left[\frac{f_6(t) - f_6(t_c)}{f_4(t)}\right]^{1 - m}$$

where $f_2(t) = \int_0^t \frac{V_p}{LG} dt$

$$f_6(t) = \int_0^t \frac{\nu_1^{\frac{m}{1-m}}}{G} f_4(t)\, f_5(t)[f_1(t)e^{f_2(t)}]^{\frac{1-2m}{1-m}} dt$$

$$f_4(t) = f_3(t)^{\frac{1}{1-m}} \mathrm{EXP}\left[\frac{2}{1-m} f_2(t)\right]$$

$$f_3(t) = \rho_1 G V_p^{H/G}$$

$$f_5 = \left(\frac{\nu'}{\nu_1}\right)^{\frac{m}{1-m}} \frac{\rho'}{\rho_1}$$

L = Bullet Location

V_p = Velocity of projectile

and prime represents evaluation at reference temperature, i.e., average of gas and wall temperature. Finally, the convective heat transfer coefficient was evaluated by use of the Chilton-Colburn analogy:

$$h = \rho u_1 C_p P_r^{-2/3} \frac{C_f}{2}$$

The important quantities of interest are momentum thickness, Reynolds' number based on momentum thickness for laminar-turbulent transition criteria, skin friction coefficient, and convective heat transfer coefficient. The listing of the computer program for evaluation of these parameters among many others is given in Appendix A. The definition of input and output is available in Appendix B

VI. UNSTEADY FREE CONVECTION AND RADIATION OUTSIDE THE GUN TUBE

The surface temperatures of the gun tube may reach as high as 2000°F at the inner surface (bore) and 1000°F at the outer surface because of the high rate of fire. The outer surface of the gun tube is surrounded by ambient air which may be at a temperature between -60°F and 160°F and may move at a velocity between zero and 60 miles an hour. Since wind velocities help to cool the gun tube faster than free convection and radiation phenomena, thermal design of a gun tube does not have to include wind velocities. The following example illustrates the order of magnitude of free convection and radiation effects.

Consider a unit length of a gun tube, 3 inches in outside diameter, with an outer surface temperature of 500°F. Let the surrounding air be at a pressure of one atmosphere and at a temperature of 70°F. The corresponding properties of air at 1 atmosphere and at a film temperature of 285°F $\left(= \frac{500 + 70}{2}\right)$ are as follows:

$\mu = 0.08$ lbm/ft-hr

$\rho = 0.0533$ lbm/ft^3

$\nu = 0.00041686$ ft^2/sec

$C_p = 0.241$ BTU/lbm - °R

$K = 0.02$ BTU/hr-ft - °R

$B = \frac{1}{T_{film}} = \frac{1}{745}$ /°F

$$\text{Grashof Number} = Gr = \frac{D^3 \rho^2 g B \Delta T}{\mu^2} = 1.66956 \times 10^6$$

$$\text{Prandtl number} = Pr = \frac{\mu C_p}{K} = 0.964$$

$$Gr.Pr = 1.60946 \times 10^6$$

This dimensionless parameter lies in the range of 10^4 to 10^9. By use of the approximate formula given by McAdams, the Nusselt number can be calculated as

$$Nu = 0.59 \, (Gr.Pr)^{1/4} = 21.015$$

Then, the heat transfer coefficient becomes

$$h = Nu\frac{K}{D} = 1.68117 \text{ BTU/hr-ft-°F}$$

The rate of heat loss per unit length of the gun tube becomes

$$Q_c = h \, (\pi D) \Delta T$$

$$= 567.7668 \text{ BTU/hr-ft}$$

The rate of energy emission from the gun tube to the surrounding air is given by

$$Q_r = \varepsilon \sigma A F_{12} T_w^4$$

where ε = Emissivity of the surface

σ = Stefan-Boltzmann Constant

A = Emission surface area

F_{12} = Radiation interchange factor

T_w = Gun tube outer surface temperature

The emissivity of the surface is assumed to be unity. This is desirable to maintain low temperatures in a gun tube. Perhaps, this could be achieved by an oxidized coating. Since the surface is convex, the interchange factor F_{12} is assumed to be unity. Also, energy absorption is assumed to be zero because of the absence of other radiating bodies.

Therefore, the net heat loss by radiation per unit length of the gun tube becomes

$$Q_r = \sigma A(T_w^4 - T_\infty^4)$$

$$= 0.171 \times 10^{-8}(\pi D)(960^4 - 530^4)$$

$$= 1039 \text{ BTU/hr-ft}$$

The boundary layers which arise by free convection also change to turbulent flow where they have reached a certain thickness. In air, the change occurs at a critical Grashof number around $Gr_x = 10^9$. This corresponds to a Reynolds' number $Re_\delta = \frac{u_e \delta}{\nu} = 550$. The numerical example cited above is still in the laminar range.

VII COMBINED ANALYSIS

The unsteady heat transfer analysis for the chosen ammunition and gun was divided into five problems in Section I. The solution to these problems was discussed in Sections II through VI. The combined analysis to fulfill the overall objective is now in order. The solutions obtained for the individual fictitious problems do not represent the solutions for the real gun tube problems because of continuous change in boundary conditions. A method of predicting and correcting these boundary conditions is desirable. Compatible boundary conditions must be introduced at interfaces between the problems.

Since Problem 1, mentioned in Section I, is a huge one, it was decided to generate ahead the thermochemical properties for various pressures for any chosen propellant. This information will be fed in as input in the form of a table for Problem 2. An interpolation scheme will be introduced to select the appropriate quantities.

To solve Problem 2, one has to know the location of boundaries, in particular, the interface between Probelms 2 and 3. Since the location of this interface is unknown and also the thickness of boundary layers, in general, is small, one can reasonably assume that the interface is the bore surface for Problem 2. However, one should include the information at the interface such as the heat loss to the gun tube and skin friction into the governing equations. These quantities will be dependent upon the development of boundary layers, i.e., Problem 3 and also the temperature of the bore surface. The outer edge of the boundary is unknown. Sometimes, the assumption that the outer edge extends to the centerline of the tube (without any loss of generality) is convenient.

Even though the boundaries are well defined for a chosen gun for Problem 4, the information at the boundaries such as heat-in due to forced convection and heat-out due to free convection and radiation (Problem 5) are lacking because these in turn are dependent upon Problem 4.

It is not clear at this time how strong the interaction or coupling is between the problems. However, it is anticipated that for any reasonable time increment, one can start with Problem 2 and proceed to the end of Problem 5 by solving one after the other and using as much of the latest information as possible. Of course, one has to use the interface boundary information from the preceding problem This procedure may be satisfactory if no strong interaction exists between the problems and if small time-increments are chosen. Otherwise, one has to iterate the procedure given above until the results converge. The possibility of decoupling Problem 2 and 5 from the procedure above should be investigated

Cookoff is still a problem with caseless ammunition and, to some extent, with conventional ammunition, if the chamber is designed for reduced weight. Accurate determination of chamber temperatures is essential to satisfy the desired reduced weight and, at the same time, to eliminate cookoff hazards.

VIII. CONCLUSIONS

A study was initiated to establish a capability to perform overall heat transfer analysis for any given dimensions of a weapon and for any specified propellant characteristics. Toward this goal, the propellant gas convective heat transfer problem was divided into five subproblems: (1) generation of thermochemical properties for any given propellant, (2) transient inviscid compressible flow through the gun barrel (core flow), (3) transient viscous compressible flow on the bore surface (boundary layers), (4) unsteady heat diffusion through the gun tube, (5) unsteady free convection and radiation outside the gun tube

The prediction of the composition of the propellant gases by chemical equilibrium chemistry for typical small arms propellants such as M18 and IMR revealed that the gases (more than 40 per cent carbon monoxide) were highly toxic. The flame temperature, density, and force of propellant differ significantly from the results reported in AMCP-706-150

The classification of flow in gun barrels is as important as the analysis on any one of the problems mentioned above. The boundary layer flow is turbulent for most of the region inside a gun barrel The chances for relaminarization are remote The rifling may increase the heat transfer as much as 25 per cent

An exact numerical solution for the unsteady, compressible, inviscid momentum and continuity equations by the method of characteristics was obtained. In general, a linear velocity gradient assumption is found to be much better than the uniform density assumption For the first portion of bullet travel (approximately 20 per cent), neither of these assumptions are satisfactory. Further work is in progress to incorporate the burning model and energy equation into the governing equations, and to obtain the complete solution by the method of characteristics.

Analytical boundary layer analysis procedure was developed based on the transient compressible boundary layer momentum integral equation. Convective heat transfer was evaluated by use of the Chilton-Colburn analogy. The input, output, and listing of computer program were given for the case of Lagrange approximation and exponential wall shear laws. An investigation to verify the validity of the Chilton-Colburn analogy (relating momentum transfer to energy transfer) of steady flow to unsteady flow is in progress.

The analysis of transient heat diffusion through single or multilayered gun tubes was developed under an In-House Laboratory Independent Research project. This is satisfactory for the present investigation. The amount of free convection and radiation is found to be of the same order of magnitude. The analysis of forced convection on the outer surface of the gun tube, because of severe wind conditions, is unnecessary due to the favorable resulting effects. The boundary layers of typical free convective flow on the outer surface of the gun tube are estimated as being of the laminar type. The analysis of the free convection and the radiation problems by explicit finite difference methods are in progress. The interaction between the problems and the overall procedure to combine all these problems was discussed.

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APPENDIX A

Listing of Computer Program

```
//MOHUTCH JOB (YYYYYYYY,5,5000),'PUNCH THAT DATA'
C
C  THIS IS THE MAIN, OR CALLING PROGRAM FOR THE SOLUTION OF THE TIME
C DEPENDENT, COMPRESSIBLE, INTEGRAL BOUNDARY LAYER MOMENTUM EQUATION
C BY THE METHOD OF CHARACTERISTICS. THE DEVELOPMENT OF THE METHOD, AND
C THE EQUATIONS EMPLOYED FOR THIS SPECIAL SOLUTION APPLICABLE TO GUN
C BOUNDARY LAYERS ARE PRESENTED IN AEROTHERM REPORT 70-18 - T.J. DAHM
C
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUB,FINTS,F1S,PRDL,RO,SFG,TW,    V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,ELD
      OMEGA#.76
C
C  READ BASIC INPUT DATA
C  DTUB   # GUN TUBE DIAMETER IN INCHES
C  ELZ    # CHAMBER LENGTH MEASURED FROM THE BREECH, IN INCHES
C  ELD    # LOCATION OF DISCONTINUITY FROM THE BREECH, IN INCHES
C  TW     # WALL TEMPERATURE, IN DEG. R
C  CP     # SPECIFIC HEAT AT CONSTANT PRESSURE,IN BTU/LBM/DEG.R
C  II     # NUMBER OF PROJECTILE POSITIONS TO BE CONSIDERED
C  PRDL=PRANDTL NUMBER
C
      READ%  5,1< DTUB,ELZ,ELD,TW,PRDL,CP,II
    1 FORMAT%6E10.3,I10<
      ELD#ELD/12.
      PRFC#%.71/PRDL<**.6667
C
C  READ AND PROCESS MORE INPUT, AND INITIALIZE INTEGRATIONS
C
      CALL INIT
      DO 21 I#1,II
C
C  READ INTERNAL BALLISTIC DATA IN CHRONOLOGICAL ORDER
C  T      # TIME IN SEC.
C  RO     # GAS DENSITY IN LBM/CU.FT.
C  T1     # GAS TEMPERATURE, IN DEG. R
C  X      # PROJECTILE POSITION RELATIVE TO INITIAL POSITION, IN INCHES
C  V      # PROJECTILE VELOCITY, IN FT./SEC
C
      READ%  5,2<T%I<,RO,T1%I<,X,V
    2 FORMAT%5E10.3<
      T%I<#T%I</1000.
      XFT%I<#%X&ELZ</12.
      IF%I-1< 10,10,11
   10 DT#0.
      GO TO 12
   11 DT#T%I<-T%I-1<
C
C  CALCULATE VISCOSITY %LBF/HR/FT<, THERMAL CONDUCTIVITY
```

```
C %BTU/HR/FT/DEG.R<, KINEMATIC VISCOSITY, AND REYNOLDS NUMBER
C
   12 VMU#.044*%T1%I</530.<**OMEGA
      VK%I<#CP*VMU/PRDL
      VNU#VMU/3600./RO
      RN#V*DTUB*RO/VMU*300.
      RNL%I<#RN*X/DTUB
C
C  FORM CHARACTERISTIC FUNCTIONS
C
      CALL CHARF
      IF(X.EQ.0.0)GO TO 211
C  HLAM  # PROJECTILE BASE HEAT TRANSFER COEFFICIENT -FROM EQUATION 113
C OF AEROTHERM REPORT 70-18, IN BTU/SEC/ SQFT/DEG.R
C
      HLAM#.0022*PRFC*CP*VMU/X*RNL%I<*12./3600./.796
      GO TO 212
  211 HLAM=0.0
  212 CONTINUE
C
C  PRINTOUT TRAJECTORY AND OTHER DATA- SAME UNITS AS INPUT
      WRITE%   6,3< T%I<,RO,T1%I<,X,V,HLAM
    3 FORMAT%10X,6%1X,E12.5<<
   21 CONTINUE
C
C  OBTAIN CHARACTERISTIC SOLUTIONS
C
      CALL GUNBL
      STOP
      END
      SUBROUTINE INIT
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUB,FINTS,F1S,PRDL,RO,SFG,TW,    V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,ELD
    9 FORMAT%3E10.3<
C
C  READ INPUT PARAMETERS M%#VMX< AND A%#AC< OF EQUATION 51, REPORT 70-18
C AND SHAPE FACTOR G. THESE ARE ALL DIMENSIONLESS
C
      READ%5,9<VMX,SFG,AC
C
C  FORM EXPONENTS THAT WILL BE USED LATER
C
      VMX1#1.-VMX
      VMX2#2.*VMX
      VMX3#VMX/VMX1
      VMX4#%1.-2.*VMX</VMX1
      VMX5#1./VMX1
```

```
C
C  INI IALIZE INTEGRATIONS
C
      F1S#0.
      FINTS#0.
      F1%1<#0.
      F2%1<#0.
      F4%1<#0.
      F6%1<#0.
      RETURN
      END
```

```
      SUBROUTINE CHARF
C
C  THIS SUBROUTINE FORMS THE CHARACTERISTIC FUNCTIONS F1,F2,F3,F4, AND
C F6, EQUATIONS 82,81,80,79, AND 122 RESPECTIVELY OF REPORT 70-18
C
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUB,FINTS,F1S,PRDL,RO,SFG,TW,   V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,FLD
   11 FORMAT%11%1XE11.4<<
C
C  OBTAIN BOUNDARY LAYER SHAPE FACTORS
C
      CALL COMBL
      IF%I-1< 34,34,33
   33 F1%I<#V/XFT%I<
      F2%I<#F2%I-1<&%F1%I</SFG&F1S</2.*DT
      EXF#SFH%I</SFG
      F3#RO*SFG*V**EXF
      F4%I<#F3**VMX5*EXP%2./VMX1*F2%I<<
      FINT#F4%I<*VNU**VMX3/SFG*%F1%I<*EXP%F2%I<<<**VMX4*REFR%I<
      F6%I<#F6%I-1<&%FINT&FINTS</2.*DT
      F1S#F1%I</SFG
      FINTS#FINT
C
C  PRINTOUT INTERMEDIATE DATA FOR REFERENCE PURPOSES
C
   34 WRITE%   6,11<T%I<,F1%I<,F2%I<,F4%I<,F6%I<,RNL%I<,XFT%I<,DOT%I<,SF
     2H%I<,SFG,REFR%I<
      RETURN
      END
```

```
      SUBROUTINE COMBL
C
C  THIS SUBROUTINE EVALUATES THE BOUNDARY LAYER SHAPE FACTORS OF
C FIGURE 9 OF REPORT 70-18, VALID ONLY FOR 1/7 POWER AND TEMPERATURE
C PROFILES. EVALUATIONS BASED ON CURVE FITS TO EXACT RESULTS.
C
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUB,FINTS,F1S,PRDL,RO,SFG,TW,    V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,ELD
      CALL PROPS
      SFG#1.2857&.0545*BETA-.0369*BETA**1.17
      SIGMA#1.-ALFA
      SFH%I<#1.2857-1.2572*SIGMA-.025*SIGMA**2.25
      DOT%I<#ALFA*%10.2857&8.06857*BETA<
      RETURN
      END
      SUBROUTINE PROPS
C
C  THIS SUBROUTINE EVALUATES THE BOUNDARY LAYER REFERENCE PROPERTY
C DATA.
C
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUB,FINTS,F1S,PRDL,RO,SFG,TW,    V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,ELD
C




C  COV#GAS COVOLUME%CUFT./LBM<. IT IS EQUAL TO 0. FOR THIS SAMPLE CASE
C
      COV#0.
      OMEG#.76
      ROCO#RO*COV
      TR#TW/T1%I<
      TRP#1.-TR
C
C  ALFA AND BETA ARE PARAMETERS  NEEDED TO EVALUATE BOUNDARY LAYER
C SHAPE FACTORS
C
      ALFA#TR&ROCO*TRP
      BETA#%TRP*%1.-ROCO<</ALFA
      TP#%TW&T1%I<</2.
      TR#TP/T1%I<
      ROR#1./%ROCO&%1.-ROCO<*TR<
      VMUR#TR**OMEG
```

```
C
C  REFR IS EQUATION 84 OF REPORT 70-18
C
      REFR%I<#ROR**VMX4*VMUR**VMX3
      RETURN
      END
      SUBROUTINE GUNBL
C
C  THIS SUBROUTINE OBTAINS THE CHARACTERISTIC SOLUTIONS AFTER ALL IN-
C TERNAL BALLISTIC DATA HAVE BEEN PROCESSED
C
      COMMON I,II
      COMMON DOT%200<,F1%200<,F2%200<,F4%200<,F6%200<,REFR%200<,RNL%200<
     2,SFH%200<,T%200<,XFT%200<,VK%200<,T1%200<,XIC%20<
      COMMON AC,ALFA,BETA,DTUR,FINTS,F1S,PRDL,RO,SFG,TW,    V,VMX,VMX1,VM
     2X2,VMX3,VMX4,VMX5,VNU,DT,ELD
    1 FORMAT%I1,E12.5<
    6 FORMAT%I10<
    7 FORMAT%8E10.3<
    8 FORMAT%4%1XF7.3<,1XF7.0,1XF7.2,3%F8.3<,1XF7.2,1XF7.5,2%1XF9.5<,2%1
     2XF7.4<<
    9 FORMAT   %1H1,44X5HTIME#,    F7.3,1X12HMILLISECONDS/30X
     220HPROJECTILE POSITION#,F7.3,1X6HINCHES//<
   10 FORMAT%  4X2HTC6X2HXC7X1HX6X3HHTC4X4HQDOT3X8HQ/%Q-DD<2X5HTHETA
     23X5HDELTA2X6HDELSTR3X3HSHR4X4HSHRT4X4HTHTT4X5HRTHTT4X4HNUTL5X2HXI<
C
C  READ INPUT FOR SOLUTION OUTPUT TIMES AND STATIONS
C     NXT#0 IF DESIRED PRINTOUT POSITIONS HAVE NOT BEEN DEFINED
C     NXT#1 IF DESIRED PRINTOUT POSITIONS HAVE BEEN DEFINED
C     NXT GREATER THAN 1 WILL CAUSE RETURN TO THE MAIN PROGRAM
C     TIME IS THE SOLUTION TIME, IN SECONDS
C
   15 READ(5,1)NXT,TIME
      IF%NXT-1<20,22,26
   20 FPRT#%AC/VMX1<**VMX5
C
C  THE FOLLOWING INPUT DEFINES PRINTOUT POSITIONS. NN IS THE NUMBER
C OF POSITONS BETWEEN THE PROJECTILE AND BREECH, AND XIC %BETWEEN 0.
C AND 1.< DEFINES THE DIMENSIONLESS ORIGIN IN SPACE OF CHARACTERISTICS
C ALONG WHICH THE OUTPUT DATA ARE EVALUATED. THESE DATA ARE READ ONLY
C IF NXT#0
C
      READ(5,6)NN
      READ%  5.7<%XIC%N<,N#1,NN<
```

```
   22 DO 24 I#1,II
C
C  FIND CHARACTERISTIC AND OTHER REQUIRED FUNCTIONS AT THE OUTPUT TIME,
C BY INTERPOLATION
C
      DELT#TIME-T%I<
      IF%DELT<23,23,24
   23 FRCC#-DELT/%T%I<-T%I-1<<
      FRC#1.-FRCC
      F1T#F1%I-1<*FRCC&F1%I<*FRC
      F2T#F2%I-1<*FRCC&F2%I<*FRC
      F4T#F4%I-1<*FRCC&F4%I<*FRC
      F6T#F6%I-1<*FRCC&F6%I<*FRC
      RNLT#RNL%I-1<*FRCC&RNL%I<*FRC
      XFTT#XFT%I-1<*FRCC&XFT%I<*FRC
      DOTT#DOT%I-1<*FRCC&DOT%I<*FRC
      SFHT#SFH%I-1<*FRCC&SFH%I<*FRC
      REFRT#REFR%I-1<*FRCC&REFR%I<*FRC
      T1T#T1%I-1<*FRCC&T1%I<*FRC
      VKT#VK%I-1<*FRCC&VK%I<*FRC
      XFTIN#XFTT*12.
      TMS#TIME*1000.
C
C  PRINT OUTPUT TIME AND PROJECTILE POSITION AT THAT TIME
C
      WRITE(6,9)TMS,XFTIN
      WRITE(6,10)
      RNLM#RNLT**VMX
      XOD#XFTIN/DTUB
      DBF#.023*%RNLT/XOD<**.8*PRDL**.4
      GNF#RNLT**VMX1*PRDL**.3333/XOD
      GO TO 25
   24 CONTINUE
   25 DO 35 N#1,NN
C
C  FIND %BETWEEN HERE AND STATEMENT 33< THE CHARACTERISTIC ORIGIN IN
C SPACE %XC<, AND THEN THE CHARACTERISTIC ORIGIN IN TIME %TC<, BASED
C ON THE DIMENSIONLESS POSITION ORIGIN OF THE OUTPUT CHARACTERISTIC, XIC
C
      XC#XFTT*%1.-XIC%N<<
C
C  CHECK FOR CHARACTERISTIC ORIGIN UPSTREAM OF THE DISCONTINUITY
C
      DX#XC-ELD
      IF%DX< 30,30,21
C
C  DETERMINE IF CHARACTERISTIC ORIGINATES IN THE CHAMBER
C
   21 DX#XC-XFT%1<
      IF%DX< 29,29,31
   29 TC#0.
      F2TC#0.
      F6TC#0.
      XFAC#XC
      GO TO 34
```

```
C
C  STATEMENT 30 THROUGH 28 DOES SPECIAL CALCULATION TO CONSIDER A DIS-
C CONTINUITY IN THE GUN BARREL ACCORDING TO BOUNDARY CONDITION 125 OF
C REPORT 70-18
C
   30 TC#%ELD-XC</ELD*TIME
      XC#ELD
      DO 28 I#1,II
      DELT#TC-T%I<
      IF%DELT< 27,27,28
   27 FRCC#-DELT/%T%I<-T%I-1<<
      FRC#1.-FRCC
      F2TC#F2%I-1<*FRCC&F2%I<*FRC
      F6TC#F6%I-1<*FRCC&F6%I<*FRC
      XFAC#XC/EXP%F2TC<
      GO TO 34
   28 CONTINUE
   31 DO 33 I#2,II
      DX#XC-XFT%I<
      IF%DX< 32,32,33
C
C  EVALUATION OF CONSTANTS OF INTEGRATION FOR CHARACTERISTICS WHICH ARE
C ORIGINATED BY THE PROJECTILE MOTION
C
   32 FRCC#-DX/%XFT%I<-XFT%I-1<<
      FRC#1.-FRCC
      F2TC#F2%I-1<*FRCC&F2%I<*FRC
      F6TC#F6%I-1<*FRCC&F6%I<*FRC
      TC#T%I-1<*FRCC&T%I<*FRC
      XFAC#XC/EXP%F2TC<
      GO TO 34
   33 CONTINUE
C
C  OBTAIN AND PRINT VALUES OF DESIRED OUTPUT DATA AT THE DESIRED
C SOLUTION TIME
C
   34 F#FPRT*XFAC**VMX4
      PSI#ABS%F*%F6T-F6TC</F4T<
      THETA#PSI**VMX1*12000.
      DELTA#DOTT*THETA
      DLSTR#SFHT*THETA
      THTT#THETA/XFTT*RNLM/12000.
      X#XFAC*EXP%F2T<
      XID#X/XFTT
      XI#1.-XID
      RTHTT#XID*THTT
      SHRT#AC**VMX5/%VMX1*RTHTT<**VMX3*REFRT
      SHR#SHRT/RNLM*1000.
      VNUTL#XID*SHRT
      XCIN#XC*12.
      TC#TC*1000.
```

```
      X#X*12.
      VNUSD#VNUTL*GNF
      HTC#VNUSD*VKT/DTUB/300.
      QDOT#HTC*%T1T-TW<
      VNUDB#DBF*XID**.8
      QDR#VNUSD/VNUDB
C
C              OUTPUT DATA
C  TC     TIME ORIGIN OF OUTPUT CHARACTERISTIC, IN MILLISEC.
C  XC     POSITION ORIGIN OF OUTPUT CHARACTERISTIC, IN INCHES
C  HTC    LOCAL HEAT TRANSFER COEFFICIENT, IN BTU/SEC/SQFT/DEG.R
C  X      CURRENT SPATIAL LOCATION OF OUTPUT CHARACTERISTIC
C  QDOT   LOCAL HEAT FLUX, IN BTU/SEC/SQFT
C  QDR    RATIO OF LOCAL HEAT FLUX TO THAT OBTAINED FROM THE DITTUS-
C BOELTER RELATION
C  THETA MOMENTUM THICKNESS, IN MILS
C  DELTA BOUNDARY LAYER THICKNESS, IN MILS
C  DLSTR DISPLACEMENT THICKNESS, IN MILS
C  SHR    1000 TIMES CF/2
C  SHRT   EQUATION 102 OF REPORT 70-18
C  THTT   EQUATION  94 OF REPORT 70-18
C  RTHTT  EQUATION 104 OF REPORT 70-18
C  VNUTL  EQUATION 106 OF REPORT 70-18
C  XI     1.-X/L
C
      WRITE%   6,8< TC,XCIN,X,HTC,QDOT,QDR,THETA,DELTA,DLSTR
     2,SHR,SHRT,THTT,RTHTT,VNUTL,XI
   35 CONTINUE
      GO TO 15
   60 STOP
   26 RETURN
      END
$ENTRY
   1.2115     2.48                   700.       1.         .421            51
   .2         1.2857     .0296
    0.        5.6144      5160.      0.         0.
    .05       5.7014      5162.      .00005     2.
    .1        5.8058      5164.      .0002      4.
    .15       5.9218      5157.      .001       7.
    .2        6.0366      5150.      .005       10.
    .25       6.1654      5132.      .015       22.
    .3        6.2953      5114.      .033       40.
    .35       6.4310      5092.      .088       69.
    .4        6.5679      5070.      .132       120.
    .45       6.7234      5035.      .222       195.
    .5        6.8800      5000.      .366       272.
```

.55	7.1038	4966.	.58	360.
.6	7.4368	4931.	.803	447.
.65	7.7824	4886.	1.18	560.
.7	8.1200	4840.	1.49	678.
.75	8.3532	4785.	1.9	785.
.8	7.8567	4730.	2.45	895.
.85	7.2349	4673.	3.	1020.
.9	6.6746	4616.	3.69	1137.
.95	6.1584	4533.	4.5	1258.
1.	5.8940	4450.	5.19	1357.
1.05	5.5552	4355.	6.05	1450.
1.1	5.2803	4260.	6.92	1537.
1.15	4.9799	4164.	7.9	1615.
1.2	4.7479	4068.	8.86	1685.
1.25	4.4741	3984.	9.92	1757.
1.3	4.2734	3900.	11.	1812.
1.35	4.0414	3822.	12.1	1868.
1.4	3.8674	3744.	13.2	1911.
1.45	3.6378	3678.	14.4	1951.
1.5	3.4185	3612.	15.6	1990.
1.55	3.2376	3556.	16.8	2022.
1.6	3.0473	3500.	18.	2050.
1.65	2.8675	3454.	19.3	2077.
1.7	2.6715	3408.	20.5	2100.
1.75	2.5068	3364.	21.8	2118.
1.8	2.3571	3320.	23.	2131.
1.85	2.2632	3276.	24.4	2148.
1.9	2.1820	3232.	25.6	2158.
1.95	2.0230	3196.	27.	2168.
2.	1.9372	3160.	28.2	2177.
2.05	1.8444	3120.	29.5	2182.
2.1	1.7539	3080.	30.8	2188.
2.15	1.6669	3040.	32.1	2192.
2.2	1.6008	3000.	33.4	2198.
2.25	1.5393	2960.	34.8	2200.
2.3	1.4813	2920.	36.1	2202.
2.35	1.4326	2880.	37.5	2204.
2.4	1.3920	2840.	38.7	2206.
2.45	1.3572	2800.	40.1	2208.
2.5	1.3282	2760.	41.4	2210.

.0005
15
.005 .02 .05 .1 .2 .3 .4 .5
.6 .7 .8 .9 .95 .98 .995

```
1   .0006
1   .0007
1   .00075
1   .0008
1   .0009
1   .001
1   .0011
1   .00125
1   .00145
1   .00162
1   .00165
1   .0017
1   .0018
1   .00195
1   .0021
1   .00225
1   .00243
$STOP
/*
```

APPENDIX B

INPUT

First Set (6E10.3, I10): 1 card

DTUB = Gun tube diameter, inches

ELZ = Chamber length measured from the breech, inches

ELD = Location of discontinuity from the breech, inches

TW = Wall temperature, °R

PRDL = Prandtl Number = $\frac{\mu C_p}{K}$

CP = Specific heat at constant pressure, BTU/lbm/°R

II = Number of projectile positions to be considered

Third Set (5E10.3): II cards

T = Time, seconds

RO = Gas density, lbm/ft^3

T1 = Gas temperature, °R

X = Projectile position relative to initial position, inches

V = Projectile velocity, ft/sec

Second Set (3E10.3). 1 card

VMX = the exponent m in Equation 52

= 0.2 for 1/7th power law velocity profile

= 0.181818 for 1/8th power law velocity profile

= 0.166667 for 1/9th power law velocity profile

SFG = Shape factor G (Equation 23)

= 1.3 for 1/7 power velocity profile

AC = the parameter (eddy viscosity) a in Equation 52

= 0.0296 for 1/7th power law velocity profile

Fourth Set (I1, E12.5):

NXT = 0 if desired printout positions have not been defined

= 1 if desired printout positions have been defined

> 1 return to the main program

TIME = Solution time, seconds

Fifth Set* (I10): 1 card

NN = Number of printout positions between the projectile and the breech

Sixth Set* (8E10.3): NN/8 cards

XIC = Dimensionless origin for characteristics along which the output is desired

OUTPUT

First Sequence (II times)

First Set (11(1X E11.4)):

T = Time, seconds

F1 = Defined in Equation 82

F2 = Defined in Equation 81

F4 = Defined in Equation 79

F6 = Defined in Equation 122

RNL = Projectile Reynolds' Number

XFT = Projectile position from breech, ft.

DOT = Defined in Equation 61, best values correspond to curve-fitted data with 1/7 power velocity profile

SFH = Defined in Equation 59, but values correspond to curve-fitted data with 1/7 power velocity profile

SFG = Defined in Equation 60, but values correspond to curve-fitted data with 1/7 power velocity profile

REFR = Defined in Equation 84

Second Set (10X, 6(1X, E12.5)):

T = Time, seconds

RO = Gas density, lbm/ft^3

T1 = Gas temperature, °R

*Fifth and sixth sets are required only if NXT =0

X = Projectile position relative to initial position, inches

V = Projectile velocity, ft/sec

HLAM = Projectile base heat transfer coefficient (Equation 113)

Second Sequence:

First Set:

TMS = Output time, seconds

XFTIN = Projectile position, inches

Second Set. Title for the following output

Third Sequence:

First Set.

TC = Time origin of output characteristics, milliseconds

XCIN = Position origin of output characteristics, inches

X = Current spatial location of output characteristic

HTC = Local heat transfer coefficient, BTU/sec-ft^2-°R

QDOT = Local heat flux, BTU/sec-ft^2

QDR = Ratio of local heat flux to that obtained from the Dittus-Boelter relation

THETA = Momentum thickness, mils

DELTA = Boundary layer thickness, mils

DLSTR = Displacement thickness, mils

SHR = Skin friction coefficient (1000 times $\frac{C_f}{2}$)

SHRT = Normalized skin friction coefficient (Equation 102)

THTT = Normalized momentum thickness (Equation 94)

RTHTT = Normalized momentum thickness Reynolds number (Equation 104)

VNUTL = Normalized Nusselt number (Equation 106)

XI = Normalized longitudinal distance from the bullet base